PQIC Technical Memorandum: Series 29, Number 3:
Arbitrary Precision Nomographic Analytical Calculation
Application Specific Integrated Circuit (APNACASIC)
Use Case #1: Combustion Engine Surveillance And Optimization

Timothy Hall

Abstract

The PQI Consulting LLC Use Cases are meant to demonstrate the utility of its products in a practical use scenario. In this document, the utility is the surveillance and optimization of the power delivered by an internal combustion engine using the Arbitrary Precision Nomographic Analytical Calculation Application Specific Integrated Circuit (APNACASIC) architecture as an embedded subsystem. The information found in this document does not indicate nor warrants that any described functionality has been (to date) actually implemented in practice – rather that such functionality is possible through the use of the APNACASIC architecture.

TABLE OF CONTENTS

1. Introduction

Consider a four-stroke internal combustion engine (such as commonly found in modern automobiles) with a fuel/oxygen-source vaporized mix that is injected into the space remaining in the combustion chamber when the piston has reached its highest point (the "combustion cap" – at top dead center), and with exhaust valves venting the post-combustion chemical species. The amount of fuel/oxygen-source placed into the combustion chamber is regulated by the injector (or by a separate control system).

Suppose there were a reserve of a special fuel additive that helped clean the carbon scoring commonly afflicting the exhaust values, as well as adjusting the pressure dynamics of the combustion progression to make the downward force on the piston more uniformly applied throughout the power stroke, thereby reducing the wear effects of piston rattle, including damage to the piston rings and linkages to the crankshaft. This relatively expensive special fuel additive must only be used sparingly, at times when most justified to have a beneficial effect, as too much use renders sub-optimal effects, and would require frequent refills only available through disruptive engine servicing.

Finally, suppose there were two sensing devices within the combustion cap and the top of the piston, respectively, that measures five performance aspects of the combustion, that, according to expert opinion, under the right combination of statistics of those measures (the "On" signal), would justify the addition (through the injectors) of the special fuel additive until a different combination of statistics of those measures (the "Off" signal) is obtained. This would conserve the use of the special fuel additive to only those combination of statistics that most justify its use.

2. Analytical Context

Label the four strokes of the engine as

1. *Intake* (exhaust values closed, piston moving away from combustion cap, fuel mixture injected into combustion chamber)
2. *Compression* (exhaust values closed, piston moving tads combustion cap, fuel mixture injector inactive; prior to ignition)
3. *Power* (exhaust values closed, piston moving away from combustion cap; after ignition)
4. *Vent* (exhaust values open, piston moving towards combustion cap)

The objective of the analysis is to find a set of independent measurements that may be used to closely and consistently predict some dependent aspect of the combustion cycle so that particular "undesirable" values of the independent measurements will prompt the fuel additive to increase (which in turn produces "optimal" values for the dependent aspect), and "desirable" values of the independent measurements will prompt less fuel additive (down to zero if justified).

Suppose the Combustion Cap Sensor were able to report four independent measurements ...

1. Minimum Temperature Of Exhaust Achieved At End Of Vent ($T_{\min}$ in $^\circ K$)
2. Maximum Temperature Of Combustants Achieved During Power ($T_{\max}$ in $^\circ K$)
3. Carbon Proportion In Combustants At Ignition (at transition from Compression to Power) (C [unitless])
4. Percent Of Combustants Still In Combustion Cap At End Of Vent (R [unitless])

... and the Piston Sensor were able to report one dependent measurement ...

1. Maximum Downward Force (perpendicular to the piston head plane) During Power ($F_{\max}$ in newtons)

... and one covariate measurement (for which there is no expert opinion consensus that this measurement has relevance to the dependent variable), namely the ...

1. Frequency Of The Crankshaft Revolution (M in Hz)

For the purposes of this use case, let $F_{\max}$ be the performance aspect we wish to optimize. The first analytical question that must be definitely answered is "do the observed values for $(M, T_{\min}, T_{\max}, C, R)$ significantly explain the observed variation of $F_{\max}$? If the answer is "No," since, say, only 65% of the variation in $F_{\max}$ may be explained by these five factors and their testable interactions (say at 90% confidence), then new independent measurements need to be defined that the Combustion Cap Sensor is capable of reporting and that do a better job of explaining the variation of $F_{\max}$. If, however, say, 90% of the observed variation in $F_{\max}$ is explained by these independent variables (at 90% confidence), then we may use their linear regression coefficients to fine-tune the analytical circumstances when (and how much) the fuel additive is needed.

However, for the purposes of this memorandum, suppose it is not known whether or not $(T_{\min}, T_{\max}, C, R, M)$ significantly explain the observed variation of $F_{\max}$. The following experimental design may be encoded into the APNACASIC control system to assess this question.

We have that $T_{\min}$, $T_{\max}$, and C are random factors since temperature and carbon proportions are a continuous measurement of kinetic energy in a system and concentration of fuel components, respectively, and the crankshaft turn speed is a continuous measurement bounded below by 0 (we assume the engine continues to run during the data collection period) with no (theoretical) upper limit. This means these independent measurements are always positive. Furthermore, we have that R is a fixed factor, observed in distinct and disjoint ranges, as is the case for M.

Suppose there were three levels for M: "Less Than 10," and "Between 10 And 20," and "More Than 20." Note that 10 Hz corresponds to 600 rpm. Further suppose that $T_{\min}$ and $T_{\max}$ are measured in $25°K$ increments (say 5 of each), and that C and R are measured in 10% increments (say 3 of each). Then a fully factorial experimental design would have an equal number of observations at each level of M, and for each value of M we have $5 \times 5 \times 3 \times 3 = 225$ observations, so that the total number of observations for this design is $3 \times 225 = 675$. Note that not every continuously-collected combination of $(T_{\min}, T_{\max}, C, R, F_{\max})$ is observed in the same manner. For example, we would need to collect 225 samples for the M-sequence ("Between 10 And 20" $\longrightarrow$ "Less Than 10" $\longrightarrow$ "More Than 20") which would require a break in observing $(T_{\min}, T_{\max}, C, R, F_{\max})$ as the engine speeds up from "Less Than 10" to "More Than 20" while going through "Between 10 And 20." Care must be taken when collecting data during the second time M is in the range "Between 10 And 20" so that there is no over-sample in that M-range. Otherwise we would have a restriction on the randomization of M; in particular, a nesting of observing $(T_{\min}, T_{\max}, C, R, F_{\max})$ within M, which would prevent a meaningful analysis of variance.[1] Therefore the collection of data must continually vary M from level to level while collecting observations for $(T_{\min}, T_{\max}, C, R, F_{\max})$ in a manner that ensures the same number of samples per level of M (225), no more (or less) than 675 samples per complete (non-nested) set of $(M, T_{\min}, T_{\max}, C, R, F_{\max})$, and at least two complete sets of $(M, T_{\min}, T_{\max}, C, R, F_{\max})$ so that there is at least one degree of freedom for the model error term.

To be specific, let $\{1, 2, 3\}$ correspond to the three levels of M (in order of "Less Than 10," and "Between 10 And 20," and "More Than 20"); let $\{1, 2, 3, 4, 5\}$ be the levels of $T_{\min}$ and $T_{\max}$, where

$$T_{\min} \in \{< 375°K,\ 375°K \text{ to } 400°K,\ 400°K \text{ to } 425°K,\ 425°K \text{ to } 450°K,\ > 450°K\}$$
$$T_{\max} \in \{< 500°K,\ 500°K \text{ to } 525°K,\ 525°K \text{ to } 550°K,\ 550°K \text{ to } 575°K,\ > 575°K\}$$

and let $\{1, 2, 3\}$ be the levels of C and R, where

$$C \in \{< 75,\ 75 \text{ to } 85,\ > 85\}$$
$$R \in \{< 15\%,\ 15\% \text{ to } 25\%,\ > 25\%\}$$

Suppose the observations of $(M, T_{\min}, T_{\max}, C, R, F_{\max})$ were collected at regular time intervals when the value of M starts at the "Less Than 10" range for 20 seconds, then it is at the "Between 10 And 20" range for 20 more seconds, then finally it is in the "More Than 20" range for another 40 seconds. Then the value of M is at the "Between 10 And 20" range for 20 seconds, and finally it is in the "Less Than 10" range for the last 20 seconds. This cycle continues as long as observations are being collected. This way the observations will be collected for the same amount of time for each value of M. This is also a realistic M-sequence since it is possible for the acceleration between each pair of these levels may be accomplished even with a small time gap between observations.

Now that the data has been collected, let κ_{ijklm} be the number of observations for the combination of the i^{th} value of M, $1 \leq i \leq 3$, the j^{th} value of U (representing $T_{\min}$), $1 \leq j \leq 5$, the k^{th} value of V (representing $T_{\max}$), $1 \leq k \leq 5$, the l^{th} value of C, $1 \leq l \leq 3$, and the m^{th} value of R, $1 \leq m \leq 3$.

Let $w = \min_{ijklm} \kappa_{ijklm}$. This value depends on the frequency with which observations are taken, i.e., more frequent observations leads to higher values of w. By restricting (either by a last-in-first-out protocol or

[1] Nesting more than one factor within a single other factor in a fully factorial analysis of variance model prevents a unique orthogonal partition of the sum of squares components, thereby making any statistical inference on any of the factors and their interactions ill-defined.

randomly down-sampling the data, if necessary) to exactly w-many observations for each combination of $ijklm$, then there will be w observations of $F_{\max}$ for each independent value combination indexed by $ijklm$. We must have $w \geq 2$; otherwise no analysis of variance is possible. If this is not the case, additional data collection must be accomplished (using the same methods as before).

For those combinations of $ijklm$ for which $\kappa_{ijklm} > w$, then either (1) the last-in $\kappa_{ijklm} - w$ should be eliminated from the analysis dataset (if timestamps of the observations are available), or (2) the data corresponding to each $ijklm$ in excess of w should be randomly down-sampled to exactly w observations. This ensures an exact fully factorial dataset analysis of variance, which is most powerful in uniformly assessing the significance of the independent terms on the variability of the dependent value. For those combinations of $ijklm$ for which $\kappa_{ijklm} < w$, then additional data collection may be needed, or two adjacent ranges of one or more variable may be consolidated to achieve equal sample sizes in each combination of $ijklm$.

All of these dataset manipulations and decision-making policies may be coded into the control system unit within an APNACASIC implementation.

After all data manipulations have been completed, including all required data re-sampling or trimming, suppose $w = 2$, so that the following table displays the $1,350$ observed values of $(M, T_{\min}, T_{\max}, C, R, F_{\max})$, where $F_{\max}$ is expressed as a RAC value as well as a IEEE-754 floating-point number.

3. Observed $(M, T_{\min}, T_{\max}, C, R, F_{\max})$ Experimental Dataset

Suppose the dataset for analysis has been collected and appropriately edited according to the methods and protocol described for this analysis. Table 1 provides a listing of dataset, including two forms for F (code for $F_{\max}$). The APNACASIC uses the RAC version of F, as well as treating all the independent variable values as $\frac{\text{integer}}{1}$ RAC values. However, the decimal floating-point version of the RAC value is more intuitive.

Table 1: APNACASIC Use Case #1 Experimental Dataset

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
1	1	1	1	1	$\frac{25401}{245}$	103.678	1	1	1	1	2	$\frac{25401}{245}$	103.678
1	1	1	1	3	$\frac{19697}{190}$	103.668	1	1	1	2	1	$\frac{12556}{123}$	102.081
1	1	1	2	2	$\frac{43202}{418}$	103.354	1	1	1	2	3	$\frac{12560}{120}$	104.667
1	1	1	3	1	$\frac{30514}{298}$	102.396	1	1	1	3	2	$\frac{14320}{139}$	103.022
1	1	1	3	3	$\frac{25401}{245}$	103.678	1	1	2	1	1	$\frac{99326}{990}$	100.329
1	1	2	1	2	$\frac{63108}{621}$	101.623	1	1	2	1	3	$\frac{22588}{222}$	101.748
1	1	2	2	1	$\frac{44333}{430}$	103.100	1	1	2	2	2	$\frac{64590}{636}$	101.557
1	1	2	2	3	$\frac{61068}{599}$	101.950	1	1	2	3	1	$\frac{61319}{606}$	101.186
1	1	2	3	2	$\frac{10523}{103}$	102.165	1	1	2	3	3	$\frac{29399}{287}$	102.436
1	1	3	1	1	$\frac{11295}{111}$	101.757	1	1	3	1	2	$\frac{26832}{263}$	102.023
1	1	3	1	3	$\frac{85060}{853}$	99.7186	1	1	3	2	1	$\frac{12320}{120}$	102.667
1	1	3	2	2	$\frac{23636}{237}$	99.7300	1	1	3	2	3	$\frac{11340}{114}$	99.4737
1	1	3	3	1	$\frac{14042}{139}$	101.022	1	1	3	3	2	$\frac{85060}{853}$	99.7186

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
1	1	3	3	3	$\frac{25913}{255}$	101.620	1	1	4	1	1	$\frac{53732}{545}$	98.5908
1	1	4	1	2	$\frac{19900}{200}$	99.5000	1	1	4	1	3	$\frac{27494}{274}$	100.343
1	1	4	2	1	$\frac{72493}{724}$	100.128	1	1	4	2	2	$\frac{19127}{190}$	100.668
1	1	4	2	3	$\frac{53732}{545}$	98.5908	1	1	4	3	1	$\frac{31337}{315}$	99.4825
1	1	4	3	2	$\frac{70277}{705}$	99.6837	1	1	4	3	3	$\frac{14089}{143}$	98.5245
1	1	5	1	1	$\frac{12064}{123}$	98.0813	1	1	5	1	2	$\frac{47889}{492}$	97.3354
1	1	5	1	3	$\frac{21971}{221}$	99.4163	1	1	5	2	1	$\frac{41490}{421}$	98.5511
1	1	5	2	2	$\frac{11073}{111}$	99.7568	1	1	5	2	3	$\frac{69572}{705}$	98.6837
1	1	5	3	1	$\frac{59501}{606}$	98.1865	1	1	5	3	2	$\frac{10112}{103}$	98.1748
1	1	5	3	3	$\frac{62348}{625}$	99.7568	1	2	1	1	1	$\frac{31136}{302}$	103.099
1	2	1	1	2	$\frac{75236}{727}$	103.488	1	2	1	1	3	$\frac{43202}{418}$	103.354
1	2	1	2	1	$\frac{22855}{221}$	103.416	1	2	1	2	2	$\frac{43202}{418}$	103.354
1	2	1	2	3	$\frac{26423}{255}$	103.620	1	2	1	3	1	$\frac{10626}{103}$	103.165
1	2	1	3	2	$\frac{27957}{273}$	102.407	1	2	1	3	3	$\frac{22810}{222}$	102.748
1	2	2	1	1	$\frac{23873}{237}$	100.730	1	2	2	1	2	$\frac{92555}{914}$	101.264
1	2	2	1	3	$\frac{38994}{383}$	101.812	1	2	2	2	1	$\frac{90970}{894}$	101.756
1	2	2	2	2	$\frac{23873}{237}$	100.730	1	2	2	2	3	$\frac{33627}{328}$	102.521
1	2	2	3	1	$\frac{27095}{263}$	103.023	1	2	2	3	2	$\frac{92555}{914}$	101.264
1	2	2	3	3	$\frac{19064}{185}$	103.049	1	2	3	1	1	$\frac{27411}{273}$	100.407
1	2	3	1	2	$\frac{90076}{894}$	100.756	1	2	3	1	3	$\frac{62487}{621}$	100.623
1	2	3	2	1	$\frac{60469}{599}$	100.950	1	2	3	2	2	$\frac{29918}{298}$	100.396
1	2	3	2	3	$\frac{85060}{853}$	99.7186	1	2	3	3	1	$\frac{18879}{185}$	102.049
1	2	3	3	2	$\frac{29918}{298}$	100.396	1	2	3	3	3	$\frac{15245}{150}$	101.633
1	2	4	1	1	$\frac{11184}{111}$	100.757	1	2	4	1	2	$\frac{47226}{468}$	100.910
1	2	4	1	3	$\frac{12862}{129}$	99.7054	1	2	4	2	1	$\frac{62973}{625}$	100.757
1	2	4	2	2	$\frac{41911}{421}$	99.5511	1	2	4	2	3	$\frac{20655}{209}$	98.8278
1	2	4	3	1	$\frac{11226}{114}$	98.4737	1	2	4	3	2	$\frac{31561}{321}$	98.3209
1	2	4	3	3	$\frac{63318}{636}$	99.5566	1	2	5	1	1	$\frac{31240}{321}$	97.3209
1	2	5	1	2	$\frac{44040}{444}$	99.1892	1	2	5	1	3	$\frac{13764}{139}$	99.0216
1	2	5	2	1	$\frac{12827}{129}$	99.4341	1	2	5	2	2	$\frac{27220}{274}$	99.3431
1	2	5	2	3	$\frac{13396}{136}$	98.5000	1	2	5	3	1	$\frac{47889}{492}$	97.3354

Continued on Next Page . . .

Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**	**M**	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**
1	2	5	3	2	$\frac{43043}{430}$	100.100	1	2	5	3	3	$\frac{27220}{274}$	99.3431
1	3	1	1	1	$\frac{33955}{328}$	103.521	1	3	1	1	2	$\frac{86766}{853}$	101.719
1	3	1	1	3	$\frac{13343}{129}$	103.434	1	3	1	2	1	$\frac{100316}{990}$	101.329
1	3	1	2	2	$\frac{100316}{990}$	101.329	1	3	1	2	3	$\frac{11517}{111}$	103.757
1	3	1	3	1	$\frac{75236}{727}$	103.488	1	3	1	3	2	$\frac{44763}{430}$	104.100
1	3	1	3	3	$\frac{12560}{120}$	104.667	1	3	2	1	1	$\frac{23873}{237}$	100.730
1	3	2	1	2	$\frac{11406}{111}$	102.757	1	3	2	1	3	$\frac{42753}{421}$	101.551
1	3	2	2	1	$\frac{32203}{321}$	100.321	1	3	2	2	2	$\frac{15907}{158}$	100.677
1	3	2	2	3	$\frac{71687}{705}$	101.684	1	3	2	3	1	$\frac{14554}{143}$	101.776
1	3	2	3	2	$\frac{12440}{120}$	103.667	1	3	2	3	3	$\frac{95299}{938}$	101.598
1	3	3	1	1	$\frac{33299}{328}$	101.521	1	3	3	1	2	$\frac{12991}{129}$	100.705
1	3	3	1	3	$\frac{22366}{222}$	100.748	1	3	3	2	1	$\frac{10420}{103}$	101.165
1	3	3	2	2	$\frac{10318}{103}$	100.175	1	3	3	2	3	$\frac{60469}{599}$	100.950
1	3	3	3	1	$\frac{30017}{297}$	101.067	1	3	3	3	2	$\frac{30532}{302}$	101.099
1	3	3	3	3	$\frac{22413}{221}$	101.416	1	3	4	1	1	$\frac{20183}{201}$	100.413
1	3	4	1	2	$\frac{15095}{150}$	100.633	1	3	4	1	3	$\frac{89182}{894}$	99.7562
1	3	4	2	1	$\frac{28825}{287}$	100.436	1	3	4	2	2	$\frac{14741}{147}$	100.279
1	3	4	2	3	$\frac{11226}{114}$	98.4737	1	3	4	3	1	$\frac{62973}{625}$	100.757
1	3	4	3	2	$\frac{14268}{143}$	99.7762	1	3	4	3	3	$\frac{29720}{297}$	100.067
1	3	5	1	1	$\frac{83354}{853}$	97.7186	1	3	5	1	2	$\frac{10214}{103}$	99.1650
1	3	5	1	3	$\frac{62682}{636}$	98.5566	1	3	5	2	1	$\frac{88833}{898}$	98.9232
1	3	5	2	2	$\frac{47889}{492}$	97.3354	1	3	5	2	3	$\frac{20446}{209}$	97.8278
1	3	5	3	1	$\frac{14594}{147}$	99.2789	1	3	5	3	2	$\frac{88833}{898}$	98.9232
1	3	5	3	3	$\frac{19700}{200}$	98.5000	1	4	1	1	1	$\frac{43174}{421}$	102.551
1	4	1	1	2	$\frac{75236}{727}$	103.488	1	4	1	1	3	$\frac{44763}{430}$	104.100
1	4	1	2	1	$\frac{12560}{120}$	104.667	1	4	1	2	2	$\frac{13940}{136}$	102.500
1	4	1	2	3	$\frac{39377}{383}$	102.812	1	4	1	3	1	$\frac{28316}{274}$	103.343
1	4	1	3	2	$\frac{11568}{114}$	101.474	1	4	1	3	3	$\frac{45816}{444}$	103.189
1	4	2	1	1	$\frac{92555}{914}$	101.264	1	4	2	1	2	$\frac{94566}{913}$	103.577
1	4	2	1	3	$\frac{14375}{143}$	100.524	1	4	2	2	1	$\frac{45372}{444}$	102.189
1	4	2	2	2	$\frac{27684}{273}$	101.407	1	4	2	2	3	$\frac{14554}{143}$	101.776

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
1	4	2	3	1	$\frac{26168}{255}$	102.620	1	4	2	3	2	$\frac{26168}{255}$	102.620
1	4	2	3	3	$\frac{14554}{143}$	101.776	1	4	3	1	1	$\frac{73782}{727}$	101.488
1	4	3	1	2	$\frac{38611}{383}$	100.812	1	4	3	1	3	$\frac{20384}{201}$	101.413
1	4	3	2	1	$\frac{30532}{302}$	101.099	1	4	3	2	2	$\frac{46173}{452}$	102.153
1	4	3	2	3	$\frac{93653}{913}$	102.577	1	4	3	3	1	$\frac{100130}{996}$	100.532
1	4	3	3	2	$\frac{73217}{724}$	101.128	1	4	3	3	3	$\frac{59764}{600}$	99.6067
1	4	4	1	1	$\frac{38228}{383}$	99.8120	1	4	4	1	2	$\frac{89731}{898}$	99.9232
1	4	4	1	3	$\frac{14268}{143}$	99.7762	1	4	4	2	1	$\frac{99134}{996}$	99.5321
1	4	4	2	2	$\frac{19127}{190}$	100.668	1	4	4	2	3	$\frac{22192}{221}$	100.416
1	4	4	3	1	$\frac{45721}{452}$	101.153	1	4	4	3	2	$\frac{62973}{625}$	100.757
1	4	4	3	3	$\frac{12956}{129}$	100.434	1	4	5	1	1	$\frac{13764}{139}$	99.0216
1	4	5	1	2	$\frac{88833}{898}$	98.9232	1	4	5	1	3	$\frac{37845}{383}$	98.8120
1	4	5	2	1	$\frac{31240}{321}$	97.3209	1	4	5	2	2	$\frac{12733}{129}$	98.7054
1	4	5	2	3	$\frac{28538}{287}$	99.4355	1	4	5	3	1	$\frac{13946}{143}$	97.5245
1	4	5	3	2	$\frac{26865}{273}$	98.4066	1	4	5	3	3	$\frac{14594}{147}$	99.2789
1	5	1	1	1	$\frac{30611}{297}$	103.067	1	5	1	1	2	$\frac{63729}{621}$	102.623
1	5	1	1	3	$\frac{100316}{990}$	101.329	1	5	1	2	1	$\frac{60964}{600}$	101.607
1	5	1	2	2	$\frac{32282}{315}$	102.483	1	5	1	2	3	$\frac{10524}{103}$	102.175
1	5	1	3	1	$\frac{15545}{150}$	103.633	1	5	1	3	2	$\frac{14320}{139}$	103.022
1	5	1	3	3	$\frac{14518}{143}$	101.524	1	5	2	1	1	$\frac{13120}{129}$	101.705
1	5	2	1	2	$\frac{44333}{430}$	103.100	1	5	2	1	3	$\frac{13804}{136}$	101.500
1	5	2	2	1	$\frac{12433}{123}$	101.081	1	5	2	2	2	$\frac{61319}{606}$	101.186
1	5	2	2	3	$\frac{27095}{263}$	103.023	1	5	2	3	1	$\frac{91527}{898}$	101.923
1	5	2	3	2	$\frac{38994}{383}$	101.812	1	5	2	3	3	$\frac{48162}{468}$	102.910
1	5	3	1	1	$\frac{90076}{894}$	100.756	1	5	3	1	2	$\frac{20384}{201}$	101.413
1	5	3	1	3	$\frac{100130}{996}$	100.532	1	5	3	2	1	$\frac{42366}{418}$	101.354
1	5	3	2	2	$\frac{73782}{727}$	101.488	1	5	3	2	3	$\frac{30017}{297}$	101.067
1	5	3	3	1	$\frac{14411}{143}$	100.776	1	5	3	3	2	$\frac{15245}{150}$	101.633
1	5	3	3	3	$\frac{29918}{298}$	100.396	1	5	4	1	1	$\frac{60107}{606}$	99.1865
1	5	4	1	2	$\frac{27138}{273}$	99.4066	1	5	4	1	3	$\frac{13532}{136}$	99.5000
1	5	4	2	1	$\frac{26917}{262}$	102.737	1	5	4	2	2	$\frac{41911}{421}$	99.5511

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**	**M**	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**
1	5	4	2	3	$\frac{72493}{724}$	100.128	1	5	4	3	1	$\frac{28825}{287}$	100.436
1	5	4	3	2	$\frac{27494}{274}$	100.343	1	5	4	3	3	$\frac{26569}{263}$	101.023
1	5	5	1	1	$\frac{46758}{468}$	99.9103	1	5	5	1	2	$\frac{18509}{185}$	100.049
1	5	5	1	3	$\frac{92485}{938}$	98.5981	1	5	5	2	1	$\frac{29928}{302}$	99.0993
1	5	5	2	2	$\frac{83354}{853}$	97.7186	1	5	5	2	3	$\frac{46758}{468}$	99.9103
1	5	5	3	1	$\frac{13946}{143}$	97.5245	1	5	5	3	2	$\frac{10214}{103}$	99.1650
1	5	5	3	3	$\frac{44040}{444}$	99.1892	1	1	1	1	1	$\frac{22855}{221}$	103.416
1	1	1	1	2	$\frac{12556}{123}$	102.081	1	1	1	1	3	$\frac{93469}{914}$	102.264
1	1	1	2	1	$\frac{19249}{185}$	104.049	1	1	1	2	2	$\frac{24110}{237}$	101.730
1	1	1	2	3	$\frac{20786}{201}$	103.413	1	1	1	3	1	$\frac{60964}{600}$	101.607
1	1	1	3	2	$\frac{49857}{492}$	101.335	1	1	1	3	3	$\frac{12556}{123}$	102.081
1	1	2	1	1	$\frac{28042}{274}$	102.343	1	1	2	1	2	$\frac{33627}{328}$	102.521
1	1	2	1	3	$\frac{20300}{200}$	101.500	1	1	2	2	1	$\frac{27095}{263}$	103.023
1	1	2	2	2	$\frac{73941}{724}$	102.128	1	1	2	2	3	$\frac{15907}{158}$	100.677
1	1	2	3	1	$\frac{54822}{545}$	100.591	1	1	2	3	2	$\frac{71687}{705}$	101.684
1	1	2	3	3	$\frac{54822}{545}$	100.591	1	1	3	1	1	$\frac{12991}{129}$	100.705
1	1	3	1	2	$\frac{12991}{129}$	100.705	1	1	3	1	3	$\frac{33299}{328}$	101.521
1	1	3	2	1	$\frac{14411}{143}$	100.776	1	1	3	2	2	$\frac{98336}{990}$	99.3293
1	1	3	2	3	$\frac{27179}{262}$	103.737	1	1	3	3	1	$\frac{15245}{150}$	101.633
1	1	3	3	2	$\frac{90629}{898}$	100.923	1	1	3	3	3	$\frac{10318}{103}$	100.175
1	1	4	1	1	$\frac{72493}{724}$	100.128	1	1	4	1	2	$\frac{13903}{139}$	100.022
1	1	4	1	3	$\frac{99134}{996}$	99.5321	1	1	4	2	1	$\frac{60107}{606}$	99.1865
1	1	4	2	2	$\frac{45721}{452}$	101.153	1	1	4	2	3	$\frac{48381}{492}$	98.3354
1	1	4	3	1	$\frac{30230}{302}$	100.099	1	1	4	3	2	$\frac{14741}{147}$	100.279
1	1	4	3	3	$\frac{70277}{705}$	99.6837	1	1	5	1	1	$\frac{23162}{237}$	97.7300
1	1	5	1	2	$\frac{89813}{914}$	98.2637	1	1	5	1	3	$\frac{12064}{123}$	98.0813
1	1	5	2	1	$\frac{21971}{221}$	99.4163	1	1	5	2	2	$\frac{62682}{636}$	98.5566
1	1	5	2	3	$\frac{14594}{147}$	99.2789	1	1	5	3	1	$\frac{10112}{103}$	98.1748
1	1	5	3	2	$\frac{41530}{418}$	99.3541	1	1	5	3	3	$\frac{24421}{245}$	99.6776
1	2	1	1	1	$\frac{16065}{158}$	101.677	1	2	1	1	2	$\frac{21282}{209}$	101.828
1	2	1	1	3	$\frac{100316}{990}$	101.329	1	2	1	2	1	$\frac{25401}{245}$	103.678

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**	**M**	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**
1	2	1	2	2	$\frac{96237}{938}$	102.598	1	2	1	2	3	$\frac{13343}{129}$	103.434
1	2	1	3	1	$\frac{27703}{262}$	105.737	1	2	1	3	2	$\frac{61925}{606}$	102.186
1	2	1	3	3	$\frac{63729}{621}$	102.623	1	2	2	1	1	$\frac{94566}{913}$	103.577
1	2	2	1	2	$\frac{21073}{209}$	100.828	1	2	2	1	3	$\frac{31967}{315}$	101.483
1	2	2	2	1	$\frac{19064}{185}$	103.049	1	2	2	2	2	$\frac{32203}{321}$	100.321
1	2	2	2	3	$\frac{63108}{621}$	101.623	1	2	2	3	1	$\frac{45372}{444}$	102.189
1	2	2	3	2	$\frac{42753}{421}$	101.551	1	2	2	3	3	$\frac{22634}{221}$	102.416
1	2	3	1	1	$\frac{10420}{103}$	101.165	1	2	3	1	2	$\frac{25913}{255}$	101.620
1	2	3	1	3	$\frac{30017}{297}$	101.067	1	2	3	2	1	$\frac{20864}{209}$	99.8278
1	2	3	2	2	$\frac{46173}{452}$	102.153	1	2	3	2	3	$\frac{62487}{621}$	100.623
1	2	3	3	1	$\frac{19317}{190}$	101.668	1	2	3	3	2	$\frac{30017}{297}$	101.067
1	2	3	3	3	$\frac{29112}{287}$	101.436	1	2	4	1	1	$\frac{59164}{600}$	98.6067
1	2	4	1	2	$\frac{15095}{150}$	100.633	1	2	4	1	3	$\frac{22144}{222}$	99.7477
1	2	4	2	1	$\frac{20183}{201}$	100.413	1	2	4	2	2	$\frac{59870}{599}$	99.9499
1	2	4	2	3	$\frac{26917}{262}$	102.737	1	2	4	3	1	$\frac{12200}{120}$	101.667
1	2	4	3	2	$\frac{25658}{255}$	100.620	1	2	4	3	3	$\frac{48381}{492}$	98.3354
1	2	5	1	1	$\frac{62348}{625}$	99.7568	1	2	5	1	2	$\frac{26865}{273}$	98.4066
1	2	5	1	3	$\frac{91827}{913}$	100.577	1	2	5	2	1	$\frac{59271}{599}$	98.9499
1	2	5	2	2	$\frac{11112}{114}$	97.4737	1	2	5	2	3	$\frac{13396}{136}$	98.5000
1	2	5	3	1	$\frac{62348}{625}$	99.7568	1	2	5	3	2	$\frac{10214}{103}$	99.1650
1	2	5	3	3	$\frac{26655}{262}$	101.737	1	3	1	1	1	$\frac{13249}{129}$	102.705
1	3	1	1	2	$\frac{93469}{914}$	102.264	1	3	1	1	3	$\frac{33955}{328}$	103.521
1	3	1	2	1	$\frac{55367}{545}$	101.591	1	3	1	2	2	$\frac{21282}{209}$	101.828
1	3	1	2	3	$\frac{32282}{315}$	102.483	1	3	1	3	1	$\frac{32282}{315}$	102.483
1	3	1	3	2	$\frac{13940}{136}$	102.500	1	3	1	3	3	$\frac{95479}{913}$	104.577
1	3	2	1	1	$\frac{61319}{606}$	101.186	1	3	2	1	2	$\frac{30216}{298}$	101.396
1	3	2	1	3	$\frac{27441}{262}$	104.737	1	3	2	2	1	$\frac{20585}{201}$	102.413
1	3	2	2	2	$\frac{91527}{898}$	101.923	1	3	2	2	3	$\frac{12440}{120}$	103.667
1	3	2	3	1	$\frac{71687}{705}$	101.684	1	3	2	3	2	$\frac{42753}{421}$	101.551
1	3	2	3	3	$\frac{46625}{452}$	103.153	1	3	3	1	1	$\frac{30017}{297}$	101.067
1	3	3	1	2	$\frac{90076}{894}$	100.756	1	3	3	1	3	$\frac{63954}{636}$	100.557

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**	**M**	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**
1	3	3	2	1	$\frac{20100}{200}$	100.500	1	3	3	2	2	$\frac{15749}{158}$	99.6772
1	3	3	2	3	$\frac{33299}{328}$	101.521	1	3	3	3	1	$\frac{11295}{111}$	101.757
1	3	3	3	2	$\frac{59764}{600}$	99.6067	1	3	3	3	3	$\frac{73782}{727}$	101.488
1	3	4	1	1	$\frac{47226}{468}$	100.910	1	3	4	1	2	$\frac{22144}{222}$	99.7477
1	3	4	1	3	$\frac{30230}{302}$	100.099	1	3	4	2	1	$\frac{53732}{545}$	98.5908
1	3	4	2	2	$\frac{92740}{913}$	101.577	1	3	4	2	3	$\frac{53732}{545}$	98.5908
1	3	4	3	1	$\frac{31337}{315}$	99.4825	1	3	4	3	2	$\frac{44484}{444}$	100.189
1	3	4	3	3	$\frac{99134}{996}$	99.5321	1	3	5	1	1	$\frac{13946}{143}$	97.5245
1	3	5	1	2	$\frac{19982}{201}$	99.4129	1	3	5	1	3	$\frac{26655}{262}$	101.737
1	3	5	2	1	$\frac{71769}{724}$	99.1285	1	3	5	2	2	$\frac{18937}{190}$	99.6684
1	3	5	2	3	$\frac{61245}{621}$	98.6232	1	3	5	3	1	$\frac{92485}{938}$	98.5981
1	3	5	3	2	$\frac{92485}{938}$	98.5981	1	3	5	3	3	$\frac{98138}{996}$	98.5321
1	4	1	1	1	$\frac{100316}{990}$	101.329	1	4	1	1	2	$\frac{15545}{150}$	103.633
1	4	1	1	3	$\frac{13343}{129}$	103.434	1	4	1	2	1	$\frac{49857}{492}$	101.335
1	4	1	2	2	$\frac{12556}{123}$	102.081	1	4	1	2	3	$\frac{65226}{636}$	102.557
1	4	1	3	1	$\frac{65226}{636}$	102.557	1	4	1	3	2	$\frac{27703}{262}$	105.737
1	4	1	3	3	$\frac{13343}{129}$	103.434	1	4	2	1	1	$\frac{90970}{894}$	101.756
1	4	2	1	2	$\frac{61068}{599}$	101.950	1	4	2	1	3	$\frac{23873}{237}$	100.730
1	4	2	2	1	$\frac{44333}{430}$	103.100	1	4	2	2	2	$\frac{48162}{468}$	102.910
1	4	2	2	3	$\frac{15907}{158}$	100.677	1	4	2	3	1	$\frac{95299}{938}$	101.598
1	4	2	3	2	$\frac{64223}{625}$	102.757	1	4	2	3	3	$\frac{73941}{724}$	102.128
1	4	3	1	1	$\frac{44928}{444}$	101.189	1	4	3	1	2	$\frac{20100}{200}$	100.500
1	4	3	1	3	$\frac{91641}{914}$	100.264	1	4	3	2	1	$\frac{19317}{190}$	101.668
1	4	3	2	2	$\frac{26832}{263}$	102.023	1	4	3	2	3	$\frac{15749}{158}$	99.6772
1	4	3	3	1	$\frac{42366}{418}$	101.354	1	4	3	3	2	$\frac{46173}{452}$	102.153
1	4	3	3	3	$\frac{60713}{606}$	100.186	1	4	4	1	1	$\frac{29620}{298}$	99.3960
1	4	4	1	2	$\frac{92740}{913}$	101.577	1	4	4	1	3	$\frac{25658}{255}$	100.620
1	4	4	2	1	$\frac{59164}{600}$	98.6067	1	4	4	2	2	$\frac{12956}{129}$	100.434
1	4	4	2	3	$\frac{31337}{315}$	99.4825	1	4	4	3	1	$\frac{27494}{274}$	100.343
1	4	4	3	2	$\frac{27138}{273}$	99.4066	1	4	4	3	3	$\frac{97346}{990}$	98.3293
1	4	5	1	1	$\frac{13946}{143}$	97.5245	1	4	5	1	2	$\frac{24421}{245}$	99.6776

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**	**M**	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**
1	4	5	1	3	$\frac{41530}{418}$	99.3541	1	4	5	2	1	$\frac{14594}{147}$	99.2789
1	4	5	2	2	$\frac{18937}{190}$	99.6684	1	4	5	2	3	$\frac{13396}{136}$	98.5000
1	4	5	3	1	$\frac{28538}{287}$	99.4355	1	4	5	3	2	$\frac{41490}{421}$	98.5511
1	4	5	3	3	$\frac{14594}{147}$	99.2789	1	5	1	1	1	$\frac{21282}{209}$	101.828
1	5	1	1	2	$\frac{10524}{103}$	102.175	1	5	1	1	3	$\frac{10524}{103}$	102.175
1	5	1	2	1	$\frac{32282}{315}$	102.483	1	5	1	2	2	$\frac{43202}{418}$	103.354
1	5	1	2	3	$\frac{102122}{996}$	102.532	1	5	1	3	1	$\frac{61667}{599}$	102.950
1	5	1	3	2	$\frac{31136}{302}$	103.099	1	5	1	3	3	$\frac{20500}{200}$	102.500
1	5	2	1	1	$\frac{71687}{705}$	101.684	1	5	2	1	2	$\frac{32203}{321}$	100.321
1	5	2	1	3	$\frac{19064}{185}$	103.049	1	5	2	2	1	$\frac{30834}{302}$	102.099
1	5	2	2	2	$\frac{95299}{938}$	101.598	1	5	2	2	3	$\frac{48162}{468}$	102.910
1	5	2	3	1	$\frac{63108}{621}$	101.623	1	5	2	3	2	$\frac{15907}{158}$	100.677
1	5	2	3	3	$\frac{14375}{143}$	100.524	1	5	3	1	1	$\frac{22366}{222}$	100.748
1	5	3	1	2	$\frac{59764}{600}$	99.6067	1	5	3	1	3	$\frac{24911}{245}$	101.678
1	5	3	2	1	$\frac{20100}{200}$	100.500	1	5	3	2	2	$\frac{26832}{263}$	102.023
1	5	3	2	3	$\frac{11295}{111}$	101.757	1	5	3	3	1	$\frac{47694}{468}$	101.910
1	5	3	3	2	$\frac{85060}{853}$	99.7186	1	5	3	3	3	$\frac{19317}{190}$	101.668
1	5	4	1	1	$\frac{29620}{298}$	99.3960	1	5	4	1	2	$\frac{11226}{114}$	98.4737
1	5	4	1	3	$\frac{72493}{724}$	100.128	1	5	4	2	1	$\frac{43473}{430}$	101.100
1	5	4	2	2	$\frac{70277}{705}$	99.6837	1	5	4	2	3	$\frac{31561}{321}$	98.3209
1	5	4	3	1	$\frac{29620}{298}$	99.3960	1	5	4	3	2	$\frac{18694}{185}$	101.049
1	5	4	3	3	$\frac{11226}{114}$	98.4737	1	5	5	1	1	$\frac{21922}{222}$	98.7477
1	5	5	1	2	$\frac{88288}{894}$	98.7562	1	5	5	1	3	$\frac{37845}{383}$	98.8120
1	5	5	2	1	$\frac{13764}{139}$	99.0216	1	5	5	2	2	$\frac{12733}{129}$	98.7054
1	5	5	2	3	$\frac{20446}{209}$	97.8278	1	5	5	3	1	$\frac{45269}{452}$	100.153
1	5	5	3	2	$\frac{72328}{727}$	99.4883	1	5	5	3	3	$\frac{28538}{287}$	99.4355
2	1	1	1	1	$\frac{104341}{971}$	107.457	2	1	1	1	2	$\frac{49539}{451}$	109.843
2	1	1	1	3	$\frac{60825}{562}$	108.230	2	1	1	2	1	$\frac{84279}{776}$	108.607
2	1	1	2	2	$\frac{73733}{664}$	111.044	2	1	1	2	3	$\frac{18620}{171}$	108.889
2	1	1	3	1	$\frac{44013}{401}$	109.758	2	1	1	3	2	$\frac{85055}{776}$	109.607
2	1	1	3	3	$\frac{85055}{776}$	109.607	2	1	2	1	1	$\frac{15859}{148}$	107.155

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
2	1	2	1	2	$\frac{13367}{121}$	110.471	2	1	2	1	3	$\frac{71054}{652}$	108.979
2	1	2	2	1	$\frac{68607}{633}$	108.384	2	1	2	2	2	$\frac{71706}{652}$	109.979
2	1	2	2	3	$\frac{18620}{171}$	108.889	2	1	2	3	1	$\frac{53568}{485}$	110.449
2	1	2	3	2	$\frac{101143}{901}$	112.256	2	1	2	3	3	$\frac{21102}{195}$	108.215
2	1	3	1	1	$\frac{60825}{562}$	108.230	2	1	3	1	2	$\frac{13745}{125}$	109.960
2	1	3	1	3	$\frac{16674}{160}$	104.212	2	1	3	2	1	$\frac{36196}{331}$	109.353
2	1	3	2	2	$\frac{105312}{971}$	108.457	2	1	3	2	3	$\frac{49990}{451}$	110.843
2	1	3	3	1	$\frac{21102}{195}$	108.215	2	1	3	3	2	$\frac{44189}{390}$	113.305
2	1	3	3	3	$\frac{69240}{633}$	109.384	2	1	4	1	1	$\frac{35865}{331}$	108.353
2	1	4	1	2	$\frac{35865}{331}$	108.353	2	1	4	1	3	$\frac{52598}{485}$	108.449
2	1	4	2	1	$\frac{61387}{562}$	109.230	2	1	4	2	2	$\frac{43971}{409}$	107.509
2	1	4	2	3	$\frac{68607}{633}$	108.384	2	1	4	3	1	$\frac{16155}{148}$	109.155
2	1	4	3	2	$\frac{49008}{451}$	108.665	2	1	4	3	3	$\frac{44013}{401}$	109.758
2	1	5	1	1	$\frac{75472}{699}$	107.971	2	1	5	1	2	$\frac{95310}{876}$	108.801
2	1	5	1	3	$\frac{71054}{652}$	108.979	2	1	5	2	1	$\frac{30529}{284}$	107.496
2	1	5	2	2	$\frac{43612}{401}$	108.758	2	1	5	2	3	$\frac{51583}{469}$	109.985
2	1	5	3	1	$\frac{15595}{143}$	109.056	2	1	5	3	2	$\frac{44380}{409}$	108.509
2	1	5	3	3	$\frac{61949}{562}$	110.230	2	2	1	1	1	$\frac{68489}{629}$	108.886
2	2	1	1	2	$\frac{43211}{401}$	107.758	2	2	1	1	3	$\frac{73069}{664}$	110.044
2	2	1	2	1	$\frac{48557}{451}$	107.665	2	2	1	2	2	$\frac{16834}{160}$	105.212
2	2	1	2	3	$\frac{25136}{234}$	107.419	2	2	1	3	1	$\frac{53568}{485}$	110.449
2	2	1	3	2	$\frac{41425}{385}$	107.597	2	2	1	3	3	$\frac{44189}{390}$	113.305
2	2	2	1	1	$\frac{18449}{171}$	107.889	2	2	2	1	2	$\frac{40655}{385}$	105.597
2	2	2	1	3	$\frac{99341}{901}$	110.256	2	2	2	2	1	$\frac{16007}{148}$	108.155
2	2	2	2	2	$\frac{25136}{234}$	107.419	2	2	2	2	3	$\frac{100242}{901}$	111.256
2	2	2	3	1	$\frac{44380}{409}$	108.509	2	2	2	3	2	$\frac{101143}{901}$	112.256
2	2	2	3	3	$\frac{69747}{629}$	110.886	2	2	3	1	1	$\frac{24902}{234}$	106.419
2	2	3	1	2	$\frac{43211}{401}$	107.758	2	2	3	1	3	$\frac{49539}{451}$	109.843
2	2	3	2	1	$\frac{71706}{652}$	109.979	2	2	3	2	2	$\frac{51583}{469}$	109.985
2	2	3	2	3	$\frac{69118}{629}$	109.886	2	2	3	3	1	$\frac{44013}{401}$	109.758
2	2	3	3	2	$\frac{97062}{876}$	110.801	2	2	3	3	3	$\frac{69240}{633}$	109.384

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
2	2	4	1	1	$\frac{95310}{876}$	108.801	2	2	4	1	2	$\frac{99341}{901}$	110.256
2	2	4	1	3	$\frac{67974}{633}$	107.384	2	2	4	2	1	$\frac{12225}{112}$	109.152
2	2	4	2	2	$\frac{15452}{143}$	108.056	2	2	4	2	3	$\frac{100242}{901}$	111.256
2	2	4	3	1	$\frac{76870}{699}$	109.971	2	2	4	3	2	$\frac{81642}{747}$	109.293
2	2	4	3	3	$\frac{69747}{629}$	110.886	2	2	5	1	1	$\frac{11453}{110}$	104.118
2	2	5	1	2	$\frac{13367}{121}$	110.471	2	2	5	1	3	$\frac{80148}{747}$	107.293
2	2	5	2	1	$\frac{16834}{160}$	105.212	2	2	5	2	2	$\frac{13488}{121}$	111.471
2	2	5	2	3	$\frac{41040}{385}$	106.597	2	2	5	3	1	$\frac{81642}{747}$	109.293
2	2	5	3	2	$\frac{52052}{469}$	110.985	2	2	5	3	3	$\frac{21102}{195}$	108.215
2	3	1	1	1	$\frac{26383}{247}$	106.814	2	3	1	1	2	$\frac{13367}{121}$	110.471
2	3	1	1	3	$\frac{40655}{385}$	105.597	2	3	1	2	1	$\frac{80895}{747}$	108.293
2	3	1	2	2	$\frac{30529}{284}$	107.496	2	3	1	2	3	$\frac{12225}{112}$	109.152
2	3	1	3	1	$\frac{12337}{112}$	110.152	2	3	1	3	2	$\frac{25370}{234}$	108.419
2	3	1	3	3	$\frac{69240}{633}$	109.384	2	3	2	1	1	$\frac{13367}{121}$	110.471
2	3	2	1	2	$\frac{43211}{401}$	107.758	2	3	2	1	3	$\frac{12113}{112}$	108.152
2	3	2	2	1	$\frac{61387}{562}$	109.230	2	3	2	2	2	$\frac{11563}{110}$	105.118
2	3	2	2	3	$\frac{105312}{971}$	108.457	2	3	2	3	1	$\frac{97062}{876}$	110.801
2	3	2	3	2	$\frac{74397}{664}$	112.044	2	3	2	3	3	$\frac{76870}{699}$	109.971
2	3	3	1	1	$\frac{104341}{971}$	107.457	2	3	3	1	2	$\frac{24902}{234}$	106.419
2	3	3	1	3	$\frac{12113}{112}$	108.152	2	3	3	2	1	$\frac{16834}{160}$	105.212
2	3	3	2	2	$\frac{16007}{148}$	108.155	2	3	3	2	3	$\frac{18620}{171}$	108.889
2	3	3	3	1	$\frac{81642}{747}$	109.293	2	3	3	3	2	$\frac{69240}{633}$	109.384
2	3	3	3	3	$\frac{106283}{971}$	109.457	2	3	4	1	1	$\frac{49539}{451}$	109.843
2	3	4	1	2	$\frac{43562}{409}$	106.509	2	3	4	1	3	$\frac{104341}{971}$	107.457
2	3	4	2	1	$\frac{13870}{125}$	110.960	2	3	4	2	2	$\frac{18620}{171}$	108.889
2	3	4	2	3	$\frac{100242}{901}$	111.256	2	3	4	3	1	$\frac{106283}{971}$	109.457
2	3	4	3	2	$\frac{18791}{171}$	109.889	2	3	4	3	3	$\frac{30813}{284}$	108.496
2	3	5	1	1	$\frac{73069}{664}$	110.044	2	3	5	1	2	$\frac{16674}{160}$	104.212
2	3	5	1	3	$\frac{48106}{451}$	106.665	2	3	5	2	1	$\frac{15452}{143}$	108.056
2	3	5	2	2	$\frac{80895}{747}$	108.293	2	3	5	2	3	$\frac{96186}{876}$	109.801
2	3	5	3	1	$\frac{16155}{148}$	109.155	2	3	5	3	2	$\frac{61949}{562}$	110.230

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**	**M**	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**
2	3	5	3	3	$\frac{15595}{143}$	109.056	2	4	1	1	1	$\frac{20712}{195}$	106.215
2	4	1	1	2	$\frac{43211}{401}$	107.758	2	4	1	1	3	$\frac{48106}{451}$	106.665
2	4	1	2	1	$\frac{18620}{171}$	108.889	2	4	1	2	2	$\frac{12225}{112}$	109.152
2	4	1	2	3	$\frac{30529}{284}$	107.496	2	4	1	3	1	$\frac{25370}{234}$	108.419
2	4	1	3	2	$\frac{44013}{401}$	109.758	2	4	1	3	3	$\frac{12337}{112}$	110.152
2	4	2	1	1	$\frac{43211}{401}$	107.758	2	4	2	1	2	$\frac{40655}{385}$	105.597
2	4	2	1	3	$\frac{26383}{247}$	106.814	2	4	2	2	1	$\frac{96186}{876}$	109.801
2	4	2	2	2	$\frac{80895}{747}$	108.293	2	4	2	2	3	$\frac{53083}{485}$	109.449
2	4	2	3	1	$\frac{15595}{143}$	109.056	2	4	2	3	2	$\frac{97062}{876}$	110.801
2	4	2	3	3	$\frac{61949}{562}$	110.230	2	4	3	1	1	$\frac{49539}{451}$	109.843
2	4	3	1	2	$\frac{68489}{629}$	108.886	2	4	3	1	3	$\frac{73069}{664}$	110.044
2	4	3	2	1	$\frac{49990}{451}$	110.843	2	4	3	2	2	$\frac{100242}{901}$	111.256
2	4	3	2	3	$\frac{71706}{652}$	109.979	2	4	3	3	1	$\frac{72358}{652}$	110.979
2	4	3	3	2	$\frac{16155}{148}$	109.155	2	4	3	3	3	$\frac{69747}{629}$	110.886
2	4	4	1	1	$\frac{20712}{195}$	106.215	2	4	4	1	2	$\frac{11453}{110}$	104.118
2	4	4	1	3	$\frac{49539}{451}$	109.843	2	4	4	2	1	$\frac{68607}{633}$	108.384
2	4	4	2	2	$\frac{80895}{747}$	108.293	2	4	4	2	3	$\frac{43971}{409}$	107.509
2	4	4	3	1	$\frac{11673}{110}$	106.118	2	4	4	3	2	$\frac{15595}{143}$	109.056
2	4	4	3	3	$\frac{30813}{284}$	108.496	2	4	5	1	1	$\frac{73069}{664}$	110.044
2	4	5	1	2	$\frac{51114}{469}$	108.985	2	4	5	1	3	$\frac{20712}{195}$	106.215
2	4	5	2	1	$\frac{76171}{699}$	108.971	2	4	5	2	2	$\frac{20907}{195}$	107.215
2	4	5	2	3	$\frac{26630}{247}$	107.814	2	4	5	3	1	$\frac{52052}{469}$	110.985
2	4	5	3	2	$\frac{16994}{160}$	106.212	2	4	5	3	3	$\frac{52052}{469}$	110.985
2	5	1	1	1	$\frac{15859}{148}$	107.155	2	5	1	1	2	$\frac{49539}{451}$	109.843
2	5	1	1	3	$\frac{13367}{121}$	110.471	2	5	1	2	1	$\frac{18620}{171}$	108.889
2	5	1	2	2	$\frac{26630}{247}$	107.814	2	5	1	2	3	$\frac{105312}{971}$	108.457
2	5	1	3	1	$\frac{49008}{451}$	108.665	2	5	1	3	2	$\frac{44189}{390}$	113.305
2	5	1	3	3	$\frac{69747}{629}$	110.886	2	5	2	1	1	$\frac{75472}{699}$	107.971
2	5	2	1	2	$\frac{83503}{776}$	107.607	2	5	2	1	3	$\frac{48106}{451}$	106.665
2	5	2	2	1	$\frac{53083}{485}$	109.449	2	5	2	2	2	$\frac{36196}{331}$	109.353
2	5	2	2	3	$\frac{25136}{234}$	107.419	2	5	2	3	1	$\frac{41425}{385}$	107.597

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
2	5	2	3	2	$\frac{13995}{125}$	111.960	2	5	2	3	3	$\frac{106283}{971}$	109.457
2	5	3	1	1	$\frac{75472}{699}$	107.971	2	5	3	1	2	$\frac{71054}{652}$	108.979
2	5	3	1	3	$\frac{24902}{234}$	106.419	2	5	3	2	1	$\frac{76171}{699}$	108.971
2	5	3	2	2	$\frac{53083}{485}$	109.449	2	5	3	2	3	$\frac{30529}{284}$	107.496
2	5	3	3	1	$\frac{30813}{284}$	108.496	2	5	3	3	2	$\frac{85055}{776}$	109.607
2	5	3	3	3	$\frac{13609}{121}$	112.471	2	5	4	1	1	$\frac{35865}{331}$	108.353
2	5	4	1	2	$\frac{35865}{331}$	108.353	2	5	4	1	3	$\frac{11453}{110}$	104.118
2	5	4	2	1	$\frac{16834}{160}$	105.212	2	5	4	2	2	$\frac{18620}{171}$	108.889
2	5	4	2	3	$\frac{13870}{125}$	110.960	2	5	4	3	1	$\frac{52052}{469}$	110.985
2	5	4	3	2	$\frac{16994}{160}$	106.212	2	5	4	3	3	$\frac{11673}{110}$	106.118
2	5	5	1	1	$\frac{80148}{747}$	107.293	2	5	5	1	2	$\frac{30245}{284}$	106.496
2	5	5	1	3	$\frac{80148}{747}$	107.293	2	5	5	2	1	$\frac{49990}{451}$	110.843
2	5	5	2	2	$\frac{16834}{160}$	105.212	2	5	5	2	3	$\frac{84279}{776}$	108.607
2	5	5	3	1	$\frac{11673}{110}$	106.118	2	5	5	3	2	$\frac{49008}{451}$	108.665
2	5	5	3	3	$\frac{49008}{451}$	108.665	2	1	1	1	1	$\frac{80148}{747}$	107.293
2	1	1	1	2	$\frac{68489}{629}$	108.886	2	1	1	1	3	$\frac{80148}{747}$	107.293
2	1	1	2	1	$\frac{26630}{247}$	107.814	2	1	1	2	2	$\frac{100242}{901}$	111.256
2	1	1	2	3	$\frac{61387}{562}$	109.230	2	1	1	3	1	$\frac{15595}{143}$	109.056
2	1	1	3	2	$\frac{97062}{876}$	110.801	2	1	1	3	3	$\frac{12337}{112}$	110.152
2	1	2	1	1	$\frac{51114}{469}$	108.985	2	1	2	1	2	$\frac{35865}{331}$	108.353
2	1	2	1	3	$\frac{30245}{284}$	106.496	2	1	2	2	1	$\frac{13488}{121}$	111.471
2	1	2	2	2	$\frac{25136}{234}$	107.419	2	1	2	2	3	$\frac{73733}{664}$	111.044
2	1	2	3	1	$\frac{101143}{901}$	112.256	2	1	2	3	2	$\frac{30813}{284}$	108.496
2	1	2	3	3	$\frac{76870}{699}$	109.971	2	1	3	1	1	$\frac{51114}{469}$	108.985
2	1	3	1	2	$\frac{95310}{876}$	108.801	2	1	3	1	3	$\frac{30245}{284}$	106.496
2	1	3	2	1	$\frac{48557}{451}$	107.665	2	1	3	2	2	$\frac{76171}{699}$	108.971
2	1	3	2	3	$\frac{73733}{664}$	111.044	2	1	3	3	1	$\frac{12337}{112}$	110.152
2	1	3	3	2	$\frac{85055}{776}$	109.607	2	1	3	3	3	$\frac{85055}{776}$	109.607
2	1	4	1	1	$\frac{60825}{562}$	108.230	2	1	4	1	2	$\frac{26383}{247}$	106.814
2	1	4	1	3	$\frac{15309}{143}$	107.056	2	1	4	2	1	$\frac{71706}{652}$	109.979
2	1	4	2	2	$\frac{43799}{390}$	112.305	2	1	4	2	3	$\frac{16834}{160}$	105.212

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
2	1	4	3	1	$\frac{13995}{125}$	111.960	2	1	4	3	2	$\frac{30813}{284}$	108.496
2	1	4	3	3	$\frac{21102}{195}$	108.215	2	1	5	1	1	$\frac{73069}{664}$	110.044
2	1	5	1	2	$\frac{43562}{409}$	106.509	2	1	5	1	3	$\frac{75472}{699}$	107.971
2	1	5	2	1	$\frac{15452}{143}$	108.056	2	1	5	2	2	$\frac{11563}{110}$	105.118
2	1	5	2	3	$\frac{43971}{409}$	107.509	2	1	5	3	1	$\frac{49008}{451}$	108.665
2	1	5	3	2	$\frac{11673}{110}$	106.118	2	1	5	3	3	$\frac{36527}{331}$	110.353
2	2	1	1	1	$\frac{43562}{409}$	106.509	2	2	1	1	2	$\frac{15859}{148}$	107.155
2	2	1	1	3	$\frac{13745}{125}$	109.960	2	2	1	2	1	$\frac{53083}{485}$	109.449
2	2	1	2	2	$\frac{20907}{195}$	107.215	2	2	1	2	3	$\frac{105312}{971}$	108.457
2	2	1	3	1	$\frac{72358}{652}$	110.979	2	2	1	3	2	$\frac{53568}{485}$	110.449
2	2	1	3	3	$\frac{76870}{699}$	109.971	2	2	2	1	1	$\frac{40655}{385}$	105.597
2	2	2	1	2	$\frac{67974}{633}$	107.384	2	2	2	1	3	$\frac{12113}{112}$	108.152
2	2	2	2	1	$\frac{51583}{469}$	109.985	2	2	2	2	2	$\frac{51583}{469}$	109.985
2	2	2	2	3	$\frac{43612}{401}$	108.758	2	2	2	3	1	$\frac{16155}{148}$	109.155
2	2	2	3	2	$\frac{26877}{247}$	108.814	2	2	2	3	3	$\frac{36527}{331}$	110.353
2	2	3	1	1	$\frac{26383}{247}$	106.814	2	2	3	1	2	$\frac{71054}{652}$	108.979
2	2	3	1	3	$\frac{24902}{234}$	106.419	2	2	3	2	1	$\frac{16834}{160}$	105.212
2	2	3	2	2	$\frac{43971}{409}$	107.509	2	2	3	2	3	$\frac{69118}{629}$	109.886
2	2	3	3	1	$\frac{53568}{485}$	110.449	2	2	3	3	2	$\frac{21102}{195}$	108.215
2	2	3	3	3	$\frac{16155}{148}$	109.155	2	2	4	1	1	$\frac{71054}{652}$	108.979
2	2	4	1	2	$\frac{35865}{331}$	108.353	2	2	4	1	3	$\frac{15859}{148}$	107.155
2	2	4	2	1	$\frac{41040}{385}$	106.597	2	2	4	2	2	$\frac{43612}{401}$	108.758
2	2	4	2	3	$\frac{51583}{469}$	109.985	2	2	4	3	1	$\frac{101143}{901}$	112.256
2	2	4	3	2	$\frac{106283}{971}$	109.457	2	2	4	3	3	$\frac{106283}{971}$	109.457
2	2	5	1	1	$\frac{83503}{776}$	107.607	2	2	5	1	2	$\frac{18449}{171}$	107.889
2	2	5	1	3	$\frac{16674}{160}$	104.212	2	2	5	2	1	$\frac{43971}{409}$	107.509
2	2	5	2	2	$\frac{25136}{234}$	107.419	2	2	5	2	3	$\frac{20907}{195}$	107.215
2	2	5	3	1	$\frac{36527}{331}$	110.353	2	2	5	3	2	$\frac{69240}{633}$	109.384
2	2	5	3	3	$\frac{26877}{247}$	108.814	2	3	1	1	1	$\frac{95310}{876}$	108.801
2	3	1	1	2	$\frac{11453}{110}$	104.118	2	3	1	1	3	$\frac{18449}{171}$	107.889
2	3	1	2	1	$\frac{48557}{451}$	107.665	2	3	1	2	2	$\frac{16834}{160}$	105.212

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
2	3	1	2	3	$\frac{13488}{121}$	111.471	2	3	1	3	1	$\frac{49008}{451}$	108.665
2	3	1	3	2	$\frac{72358}{652}$	110.979	2	3	1	3	3	$\frac{50441}{451}$	111.843
2	3	2	1	1	$\frac{60825}{562}$	108.230	2	3	2	1	2	$\frac{43562}{409}$	106.509
2	3	2	1	3	$\frac{68489}{629}$	108.886	2	3	2	2	1	$\frac{43799}{390}$	112.305
2	3	2	2	2	$\frac{13870}{125}$	110.960	2	3	2	2	3	$\frac{12225}{112}$	109.152
2	3	2	3	1	$\frac{106283}{971}$	109.457	2	3	2	3	2	$\frac{30813}{284}$	108.496
2	3	2	3	3	$\frac{13995}{125}$	111.960	2	3	3	1	1	$\frac{43562}{409}$	106.509
2	3	3	1	2	$\frac{11453}{110}$	104.118	2	3	3	1	3	$\frac{15309}{143}$	107.056
2	3	3	2	1	$\frac{13870}{125}$	110.960	2	3	3	2	2	$\frac{68607}{633}$	108.384
2	3	3	2	3	$\frac{25136}{234}$	107.419	2	3	3	3	1	$\frac{12337}{112}$	110.152
2	3	3	3	2	$\frac{97062}{876}$	110.801	2	3	3	3	3	$\frac{18791}{171}$	109.889
2	3	4	1	1	$\frac{73069}{664}$	110.044	2	3	4	1	2	$\frac{51114}{469}$	108.985
2	3	4	1	3	$\frac{80148}{747}$	107.293	2	3	4	2	1	$\frac{16834}{160}$	105.212
2	3	4	2	2	$\frac{15452}{143}$	108.056	2	3	4	2	3	$\frac{25136}{234}$	107.419
2	3	4	3	1	$\frac{13995}{125}$	111.960	2	3	4	3	2	$\frac{85055}{776}$	109.607
2	3	4	3	3	$\frac{44380}{409}$	108.509	2	3	5	1	1	$\frac{71054}{652}$	108.979
2	3	5	1	2	$\frac{24902}{234}$	106.419	2	3	5	1	3	$\frac{68489}{629}$	108.886
2	3	5	2	1	$\frac{96186}{876}$	109.801	2	3	5	2	2	$\frac{11563}{110}$	105.118
2	3	5	2	3	$\frac{73733}{664}$	111.044	2	3	5	3	1	$\frac{13995}{125}$	111.960
2	3	5	3	2	$\frac{41425}{385}$	107.597	2	3	5	3	3	$\frac{21102}{195}$	108.215
2	4	1	1	1	$\frac{35865}{331}$	108.353	2	4	1	1	2	$\frac{60825}{562}$	108.230
2	4	1	1	3	$\frac{52598}{485}$	108.449	2	4	1	2	1	$\frac{51583}{469}$	109.985
2	4	1	2	2	$\frac{61387}{562}$	109.230	2	4	1	2	3	$\frac{26630}{247}$	107.814
2	4	1	3	1	$\frac{18791}{171}$	109.889	2	4	1	3	2	$\frac{44189}{390}$	113.305
2	4	1	3	3	$\frac{26877}{247}$	108.814	2	4	2	1	1	$\frac{40655}{385}$	105.597
2	4	2	1	2	$\frac{43409}{390}$	111.305	2	4	2	1	3	$\frac{40655}{385}$	105.597
2	4	2	2	1	$\frac{13488}{121}$	111.471	2	4	2	2	2	$\frac{15452}{143}$	108.056
2	4	2	2	3	$\frac{43612}{401}$	108.758	2	4	2	3	1	$\frac{50441}{451}$	111.843
2	4	2	3	2	$\frac{81642}{747}$	109.293	2	4	2	3	3	$\frac{74397}{664}$	112.044
2	4	3	1	1	$\frac{52598}{485}$	108.449	2	4	3	1	2	$\frac{99341}{901}$	110.256
2	4	3	1	3	$\frac{104341}{971}$	107.457	2	4	3	2	1	$\frac{12225}{112}$	109.152

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
2	4	3	2	2	$\frac{84279}{776}$	108.607	2	4	3	2	3	$\frac{69118}{629}$	109.886
2	4	3	3	1	$\frac{50441}{451}$	111.843	2	4	3	3	2	$\frac{50441}{451}$	111.843
2	4	3	3	3	$\frac{21102}{195}$	108.215	2	4	4	1	1	$\frac{99341}{901}$	110.256
2	4	4	1	2	$\frac{26383}{247}$	106.814	2	4	4	1	3	$\frac{95310}{876}$	108.801
2	4	4	2	1	$\frac{30529}{284}$	107.496	2	4	4	2	2	$\frac{20907}{195}$	107.215
2	4	4	2	3	$\frac{15452}{143}$	108.056	2	4	4	3	1	$\frac{76870}{699}$	109.971
2	4	4	3	2	$\frac{16155}{148}$	109.155	2	4	4	3	3	$\frac{36527}{331}$	110.353
2	4	5	1	1	$\frac{67974}{633}$	107.384	2	4	5	1	2	$\frac{60825}{562}$	108.230
2	4	5	1	3	$\frac{48106}{451}$	106.665	2	4	5	2	1	$\frac{15452}{143}$	108.056
2	4	5	2	2	$\frac{16007}{148}$	108.155	2	4	5	2	3	$\frac{84279}{776}$	108.607
2	4	5	3	1	$\frac{12337}{112}$	110.152	2	4	5	3	2	$\frac{53568}{485}$	110.449
2	4	5	3	3	$\frac{69240}{633}$	109.384	2	5	1	1	1	$\frac{26383}{247}$	106.814
2	5	1	1	2	$\frac{67974}{633}$	107.384	2	5	1	1	3	$\frac{15859}{148}$	107.155
2	5	1	2	1	$\frac{76171}{699}$	108.971	2	5	1	2	2	$\frac{43612}{401}$	108.758
2	5	1	2	3	$\frac{48557}{451}$	107.665	2	5	1	3	1	$\frac{44189}{390}$	113.305
2	5	1	3	2	$\frac{18791}{171}$	109.889	2	5	1	3	3	$\frac{53568}{485}$	110.449
2	5	2	1	1	$\frac{13745}{125}$	109.960	2	5	2	1	2	$\frac{40655}{385}$	105.597
2	5	2	1	3	$\frac{43409}{390}$	111.305	2	5	2	2	1	$\frac{13488}{121}$	111.471
2	5	2	2	2	$\frac{84279}{776}$	108.607	2	5	2	2	3	$\frac{69118}{629}$	109.886
2	5	2	3	1	$\frac{44189}{390}$	113.305	2	5	2	3	2	$\frac{74397}{664}$	112.044
2	5	2	3	3	$\frac{44380}{409}$	108.509	2	5	3	1	1	$\frac{15309}{143}$	107.056
2	5	3	1	2	$\frac{11453}{110}$	104.118	2	5	3	1	3	$\frac{26383}{247}$	106.814
2	5	3	2	1	$\frac{43799}{390}$	112.305	2	5	3	2	2	$\frac{71706}{652}$	109.979
2	5	3	2	3	$\frac{41040}{385}$	106.597	2	5	3	3	1	$\frac{85055}{776}$	109.607
2	5	3	3	2	$\frac{44189}{390}$	113.305	2	5	3	3	3	$\frac{11673}{110}$	106.118
2	5	4	1	1	$\frac{52598}{485}$	108.449	2	5	4	1	2	$\frac{99341}{901}$	110.256
2	5	4	1	3	$\frac{68489}{629}$	108.886	2	5	4	2	1	$\frac{41040}{385}$	106.597
2	5	4	2	2	$\frac{13488}{121}$	111.471	2	5	4	2	3	$\frac{49990}{451}$	110.843
2	5	4	3	1	$\frac{13995}{125}$	111.960	2	5	4	3	2	$\frac{36527}{331}$	110.353
2	5	4	3	3	$\frac{13995}{125}$	111.960	2	5	5	1	1	$\frac{73069}{664}$	110.044
2	5	5	1	2	$\frac{43409}{390}$	111.305	2	5	5	1	3	$\frac{60825}{562}$	108.230

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
2	5	5	2	1	$\frac{18620}{171}$	108.889	2	5	5	2	2	$\frac{43799}{390}$	112.305
2	5	5	2	3	$\frac{13488}{121}$	111.471	2	5	5	3	1	$\frac{13609}{121}$	112.471
2	5	5	3	2	$\frac{97062}{876}$	110.801	2	5	5	3	3	$\frac{76870}{699}$	109.971
3	1	1	1	1	$\frac{15247}{133}$	114.639	3	1	1	1	2	$\frac{15141}{130}$	116.469
3	1	1	1	3	$\frac{58594}{536}$	109.317	3	1	1	2	1	$\frac{59666}{536}$	111.317
3	1	1	2	2	$\frac{43658}{385}$	113.397	3	1	1	2	3	$\frac{48984}{422}$	116.076
3	1	1	3	1	$\frac{27291}{234}$	116.628	3	1	1	3	2	$\frac{21035}{175}$	120.200
3	1	1	3	3	$\frac{28954}{247}$	117.223	3	1	2	1	1	$\frac{55096}{469}$	117.475
3	1	2	1	2	$\frac{38571}{331}$	116.529	3	1	2	1	3	$\frac{26589}{234}$	113.628
3	1	2	2	1	$\frac{108928}{902}$	120.763	3	1	2	2	2	$\frac{44043}{385}$	114.397
3	1	2	2	3	$\frac{47954}{390}$	122.959	3	1	2	3	1	$\frac{22254}{189}$	117.746
3	1	2	3	2	$\frac{15912}{133}$	119.639	3	1	2	3	3	$\frac{56972}{469}$	121.475
3	1	3	1	1	$\frac{15513}{133}$	116.639	3	1	3	1	2	$\frac{90335}{776}$	116.411
3	1	3	1	3	$\frac{55565}{469}$	118.475	3	1	3	2	1	$\frac{49828}{422}$	118.076
3	1	3	2	2	$\frac{109830}{902}$	121.763	3	1	3	2	3	$\frac{91887}{776}$	118.411
3	1	3	3	1	$\frac{13577}{112}$	121.223	3	1	3	3	2	$\frac{15921}{130}$	122.469
3	1	3	3	3	$\frac{31110}{251}$	123.944	3	1	4	1	1	$\frac{12339}{110}$	112.173
3	1	4	1	2	$\frac{47954}{390}$	122.959	3	1	4	1	3	$\frac{13241}{112}$	118.223
3	1	4	2	1	$\frac{13465}{112}$	120.223	3	1	4	2	2	$\frac{22878}{195}$	117.323
3	1	4	2	3	$\frac{39895}{331}$	120.529	3	1	4	3	1	$\frac{16178}{133}$	121.639
3	1	4	3	2	$\frac{101948}{811}$	125.707	3	1	4	3	3	$\frac{13889}{114}$	121.833
3	1	5	1	1	$\frac{16815}{143}$	117.587	3	1	5	1	2	$\frac{21035}{175}$	120.200
3	1	5	1	3	$\frac{15779}{133}$	118.639	3	1	5	2	1	$\frac{45198}{385}$	117.397
3	1	5	2	2	$\frac{45198}{385}$	117.397	3	1	5	2	3	$\frac{17101}{143}$	119.587
3	1	5	3	1	$\frac{78203}{629}$	124.329	3	1	5	3	2	$\frac{109140}{902}$	120.998
3	1	5	3	3	$\frac{83707}{664}$	126.065	3	2	1	1	1	$\frac{16243}{143}$	113.587
3	2	1	1	2	$\frac{16977}{149}$	113.940	3	2	1	1	3	$\frac{16977}{149}$	113.940
3	2	1	2	1	$\frac{57288}{495}$	115.733	3	2	1	2	2	$\frac{26823}{234}$	114.628
3	2	1	2	3	$\frac{17275}{149}$	115.940	3	2	1	3	1	$\frac{44428}{385}$	115.397
3	2	1	3	2	$\frac{99515}{811}$	122.707	3	2	1	3	3	$\frac{109830}{902}$	121.763
3	2	2	1	1	$\frac{111846}{971}$	115.186	3	2	2	1	2	$\frac{31057}{273}$	113.762

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
3	2	2	1	3	$\frac{13205}{114}$	115.833	3	2	2	2	1	$\frac{49406}{422}$	117.076
3	2	2	2	2	$\frac{15646}{133}$	117.639	3	2	2	2	3	$\frac{13433}{114}$	117.833
3	2	2	3	1	$\frac{29201}{247}$	118.223	3	2	2	3	2	$\frac{13661}{114}$	119.833
3	2	2	3	3	$\frac{106434}{902}$	117.998	3	2	3	1	1	$\frac{13129}{112}$	117.223
3	2	3	1	2	$\frac{12229}{110}$	111.173	3	2	3	1	3	$\frac{38902}{331}$	117.529
3	2	3	2	1	$\frac{91887}{776}$	118.411	3	2	3	2	2	$\frac{16815}{143}$	117.587
3	2	3	2	3	$\frac{55434}{466}$	118.957	3	2	3	3	1	$\frac{16045}{133}$	120.639
3	2	3	3	2	$\frac{15921}{130}$	122.469	3	2	3	3	3	$\frac{15921}{130}$	122.469
3	2	4	1	1	$\frac{36416}{300}$	121.387	3	2	4	1	2	$\frac{15646}{133}$	117.639
3	2	4	1	3	$\frac{91111}{776}$	117.411	3	2	4	2	1	$\frac{76316}{629}$	121.329
3	2	4	2	2	$\frac{29201}{247}$	118.223	3	2	4	2	3	$\frac{12559}{110}$	114.173
3	2	4	3	1	$\frac{51094}{422}$	121.076	3	2	4	3	2	$\frac{16051}{130}$	123.469
3	2	4	3	3	$\frac{108238}{902}$	119.998	3	2	5	1	1	$\frac{44428}{385}$	115.397
3	2	5	1	2	$\frac{55434}{466}$	118.957	3	2	5	1	3	$\frac{17573}{149}$	117.940
3	2	5	2	1	$\frac{27759}{234}$	118.628	3	2	5	2	2	$\frac{116701}{971}$	120.186
3	2	5	2	3	$\frac{32422}{273}$	118.762	3	2	5	3	1	$\frac{29942}{247}$	121.223
3	2	5	3	2	$\frac{19911}{161}$	123.671	3	2	5	3	3	$\frac{16181}{130}$	124.469
3	3	1	1	1	$\frac{78395}{664}$	118.065	3	3	1	1	2	$\frac{101924}{902}$	112.998
3	3	1	1	3	$\frac{20335}{175}$	116.200	3	3	1	2	1	$\frac{54502}{466}$	116.957
3	3	1	2	2	$\frac{17275}{149}$	115.940	3	3	1	2	3	$\frac{18945}{161}$	117.671
3	3	1	3	1	$\frac{48344}{390}$	123.959	3	3	1	3	2	$\frac{16815}{143}$	117.587
3	3	1	3	3	$\frac{22065}{189}$	116.746	3	3	2	1	1	$\frac{79059}{664}$	119.065
3	3	2	1	2	$\frac{47174}{390}$	120.959	3	3	2	1	3	$\frac{16386}{143}$	114.587
3	3	2	2	1	$\frac{44043}{385}$	114.397	3	3	2	2	2	$\frac{49406}{422}$	117.076
3	3	2	2	3	$\frac{49406}{422}$	117.076	3	3	2	3	1	$\frac{19428}{161}$	120.671
3	3	2	3	2	$\frac{58773}{495}$	118.733	3	3	2	3	3	$\frac{32149}{273}$	117.762
3	3	3	1	1	$\frac{108026}{902}$	119.763	3	3	3	1	2	$\frac{26823}{234}$	114.628
3	3	3	1	3	$\frac{30106}{251}$	119.944	3	3	3	2	1	$\frac{13353}{112}$	119.223
3	3	3	2	2	$\frac{22065}{189}$	116.746	3	3	3	2	3	$\frac{105532}{902}$	116.998
3	3	3	3	1	$\frac{82379}{664}$	124.065	3	3	3	3	2	$\frac{22691}{187}$	121.342
3	3	3	3	3	$\frac{16045}{133}$	120.639	3	3	4	1	1	$\frac{17424}{149}$	116.940

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
3	3	4	1	2	$\frac{60202}{536}$	112.317	3	3	4	1	3	$\frac{20860}{175}$	119.200
3	3	4	2	1	$\frac{22254}{189}$	117.746	3	3	4	2	2	$\frac{39895}{331}$	120.529
3	3	4	2	3	$\frac{12559}{110}$	114.173	3	3	4	3	1	$\frac{19750}{161}$	122.671
3	3	4	3	2	$\frac{117672}{971}$	121.186	3	3	4	3	3	$\frac{94215}{776}$	121.411
3	3	5	1	1	$\frac{31876}{273}$	116.762	3	3	5	1	2	$\frac{16815}{143}$	117.587
3	3	5	1	3	$\frac{58278}{495}$	117.733	3	3	5	2	1	$\frac{16045}{133}$	120.639
3	3	5	2	2	$\frac{61810}{536}$	115.317	3	3	5	2	3	$\frac{23073}{195}$	118.323
3	3	5	3	1	$\frac{13801}{112}$	123.223	3	3	5	3	2	$\frac{23463}{195}$	120.323
3	3	5	3	3	$\frac{51516}{422}$	122.076	3	4	1	1	1	$\frac{88783}{776}$	114.411
3	4	1	1	2	$\frac{56298}{495}$	113.733	3	4	1	1	3	$\frac{13091}{114}$	114.833
3	4	1	2	1	$\frac{13319}{114}$	116.833	3	4	1	2	2	$\frac{20685}{175}$	118.200
3	4	1	2	3	$\frac{17275}{149}$	115.940	3	4	1	3	1	$\frac{91887}{776}$	118.411
3	4	1	3	2	$\frac{39564}{331}$	119.529	3	4	1	3	3	$\frac{21035}{175}$	120.200
3	4	2	1	1	$\frac{56793}{495}$	114.733	3	4	2	1	2	$\frac{13205}{114}$	115.833
3	4	2	1	3	$\frac{15271}{130}$	117.469	3	4	2	2	1	$\frac{12339}{110}$	112.173
3	4	2	2	2	$\frac{80387}{664}$	121.065	3	4	2	2	3	$\frac{108928}{902}$	120.763
3	4	2	3	1	$\frac{19428}{161}$	120.671	3	4	2	3	2	$\frac{92663}{776}$	119.411
3	4	2	3	3	$\frac{76316}{629}$	121.329	3	4	3	1	1	$\frac{22293}{195}$	114.323
3	4	3	1	2	$\frac{103728}{902}$	114.998	3	4	3	1	3	$\frac{38902}{331}$	117.529
3	4	3	2	1	$\frac{105532}{902}$	116.998	3	4	3	2	2	$\frac{22317}{187}$	119.342
3	4	3	2	3	$\frac{22317}{187}$	119.342	3	4	3	3	1	$\frac{37316}{300}$	124.387
3	4	3	3	2	$\frac{76945}{629}$	122.329	3	4	3	3	3	$\frac{13577}{112}$	121.223
3	4	4	1	1	$\frac{22130}{187}$	118.342	3	4	4	1	2	$\frac{47954}{390}$	122.959
3	4	4	1	3	$\frac{22130}{187}$	118.342	3	4	4	2	1	$\frac{22878}{195}$	117.323
3	4	4	2	2	$\frac{100326}{811}$	123.707	3	4	4	2	3	$\frac{29201}{247}$	118.223
3	4	4	3	1	$\frac{37616}{300}$	125.387	3	4	4	3	2	$\frac{22878}{187}$	122.342
3	4	4	3	3	$\frac{19750}{161}$	122.671	3	4	5	1	1	$\frac{36716}{300}$	122.387
3	4	5	1	2	$\frac{56503}{469}$	120.475	3	4	5	1	3	$\frac{49828}{422}$	118.076
3	4	5	2	1	$\frac{32422}{273}$	118.762	3	4	5	2	2	$\frac{50672}{422}$	120.076
3	4	5	2	3	$\frac{15921}{130}$	122.469	3	4	5	3	1	$\frac{113438}{902}$	125.763
3	4	5	3	2	$\frac{19911}{161}$	123.671	3	4	5	3	3	$\frac{29942}{247}$	121.223

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**	**M**	**U**	**V**	**C**	**R**	**F (RAC)**	**F (float)**
3	5	1	1	1	$\frac{15141}{130}$	116.469	3	5	1	1	2	$\frac{27966}{247}$	113.223
3	5	1	1	3	$\frac{29604}{251}$	117.944	3	5	1	2	1	$\frac{12229}{110}$	111.173
3	5	1	2	2	$\frac{112817}{971}$	116.186	3	5	1	2	3	$\frac{97893}{811}$	120.707
3	5	1	3	1	$\frac{105532}{902}$	116.998	3	5	1	3	2	$\frac{36716}{300}$	122.387
3	5	1	3	3	$\frac{75687}{629}$	120.329	3	5	2	1	1	$\frac{22098}{195}$	113.323
3	5	2	1	2	$\frac{31057}{273}$	113.762	3	5	2	1	3	$\frac{102826}{902}$	113.998
3	5	2	2	1	$\frac{30357}{251}$	120.944	3	5	2	2	2	$\frac{16672}{143}$	116.587
3	5	2	2	3	$\frac{15531}{130}$	119.469	3	5	2	3	1	$\frac{110732}{902}$	122.763
3	5	2	3	2	$\frac{56972}{469}$	121.475	3	5	2	3	3	$\frac{115730}{971}$	119.186
3	5	3	1	1	$\frac{112817}{971}$	116.186	3	5	3	1	2	$\frac{57288}{495}$	115.733
3	5	3	1	3	$\frac{13319}{114}$	116.833	3	5	3	2	1	$\frac{60738}{536}$	113.317
3	5	3	2	2	$\frac{60738}{536}$	113.317	3	5	3	2	3	$\frac{114759}{971}$	118.186
3	5	3	3	1	$\frac{82379}{664}$	124.065	3	5	3	3	2	$\frac{37316}{300}$	124.387
3	5	3	3	3	$\frac{56366}{466}$	120.957	3	5	4	1	1	$\frac{21876}{189}$	115.746
3	5	4	1	2	$\frac{39233}{331}$	118.529	3	5	4	1	3	$\frac{49406}{422}$	117.076
3	5	4	2	1	$\frac{55900}{466}$	119.957	3	5	4	2	2	$\frac{100326}{811}$	123.707
3	5	4	2	3	$\frac{29201}{247}$	118.223	3	5	4	3	1	$\frac{27993}{234}$	119.628
3	5	4	3	2	$\frac{16178}{133}$	121.639	3	5	4	3	3	$\frac{37616}{300}$	125.387
3	5	5	1	1	$\frac{99515}{811}$	122.707	3	5	5	1	2	$\frac{48344}{390}$	123.959
3	5	5	1	3	$\frac{75687}{629}$	120.329	3	5	5	2	1	$\frac{57441}{469}$	122.475
3	5	5	2	2	$\frac{22443}{189}$	118.746	3	5	5	2	3	$\frac{101137}{811}$	124.707
3	5	5	3	1	$\frac{32968}{273}$	120.762	3	5	5	3	2	$\frac{49904}{390}$	127.959
3	5	5	3	3	$\frac{40888}{331}$	123.529	3	1	1	1	1	$\frac{101924}{902}$	112.998
3	1	1	1	2	$\frac{106222}{902}$	117.763	3	1	1	1	3	$\frac{20335}{175}$	116.200
3	1	1	2	1	$\frac{20685}{175}$	118.200	3	1	1	2	2	$\frac{26823}{234}$	114.628
3	1	1	2	3	$\frac{18945}{161}$	117.671	3	1	1	3	1	$\frac{16815}{143}$	117.587
3	1	1	3	2	$\frac{81051}{664}$	122.065	3	1	1	3	3	$\frac{44428}{385}$	115.397
3	1	2	1	1	$\frac{15271}{130}$	117.469	3	1	2	1	2	$\frac{97082}{811}$	119.707
3	1	2	1	3	$\frac{111846}{971}$	115.186	3	1	2	2	1	$\frac{12339}{110}$	112.173
3	1	2	2	2	$\frac{22130}{187}$	118.342	3	1	2	2	3	$\frac{30357}{251}$	120.944
3	1	2	3	1	$\frac{22504}{187}$	120.342	3	1	2	3	2	$\frac{39895}{331}$	120.529

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
3	1	2	3	3	$\frac{22504}{187}$	120.342	3	1	3	1	1	$\frac{43658}{385}$	113.397
3	1	3	1	2	$\frac{103728}{902}$	114.998	3	1	3	1	3	$\frac{90335}{776}$	116.411
3	1	3	2	1	$\frac{22065}{189}$	116.746	3	1	3	2	2	$\frac{36716}{300}$	122.387
3	1	3	2	3	$\frac{13353}{112}$	119.223	3	1	3	3	1	$\frac{32422}{273}$	118.762
3	1	3	3	2	$\frac{82379}{664}$	124.065	3	1	3	3	3	$\frac{59268}{495}$	119.733
3	1	4	1	1	$\frac{54968}{466}$	117.957	3	1	4	1	2	$\frac{31603}{273}$	115.762
3	1	4	1	3	$\frac{20860}{175}$	119.200	3	1	4	2	1	$\frac{56972}{469}$	121.475
3	1	4	2	2	$\frac{110732}{902}$	122.763	3	1	4	2	3	$\frac{39895}{331}$	120.529
3	1	4	3	1	$\frac{77574}{629}$	123.329	3	1	4	3	2	$\frac{22632}{189}$	119.746
3	1	4	3	3	$\frac{13689}{112}$	122.223	3	1	5	1	1	$\frac{27291}{234}$	116.628
3	1	5	1	2	$\frac{22317}{187}$	119.342	3	1	5	1	3	$\frac{60738}{536}$	113.317
3	1	5	2	1	$\frac{45198}{385}$	117.397	3	1	5	2	2	$\frac{49124}{390}$	125.959
3	1	5	2	3	$\frac{56366}{466}$	120.957	3	1	5	3	1	$\frac{40888}{331}$	123.529
3	1	5	3	2	$\frac{51516}{422}$	122.076	3	1	5	3	3	$\frac{102759}{811}$	126.707
3	2	1	1	1	$\frac{54627}{469}$	116.475	3	2	1	1	2	$\frac{110875}{971}$	114.186
3	2	1	1	3	$\frac{12905}{112}$	115.223	3	2	1	2	1	$\frac{28460}{247}$	115.223
3	2	1	2	2	$\frac{108026}{902}$	119.763	3	2	1	2	3	$\frac{28460}{247}$	115.223
3	2	1	3	1	$\frac{114759}{971}$	118.186	3	2	1	3	2	$\frac{58278}{495}$	117.733
3	2	1	3	3	$\frac{27291}{234}$	116.628	3	2	2	1	1	$\frac{17126}{149}$	114.940
3	2	2	1	2	$\frac{54036}{466}$	115.957	3	2	2	1	3	$\frac{31057}{273}$	113.762
3	2	2	2	1	$\frac{28707}{247}$	116.223	3	2	2	2	2	$\frac{22488}{195}$	115.323
3	2	2	2	3	$\frac{91111}{776}$	117.411	3	2	2	3	1	$\frac{12559}{110}$	114.173
3	2	2	3	2	$\frac{19428}{161}$	120.671	3	2	2	3	3	$\frac{106434}{902}$	117.998
3	2	3	1	1	$\frac{57288}{495}$	115.733	3	2	3	1	2	$\frac{74429}{629}$	118.329
3	2	3	1	3	$\frac{26823}{234}$	114.628	3	2	3	2	1	$\frac{28954}{247}$	117.223
3	2	3	2	2	$\frac{36716}{300}$	122.387	3	2	3	2	3	$\frac{75687}{629}$	120.329
3	2	3	3	1	$\frac{17101}{143}$	119.587	3	2	3	3	2	$\frac{45198}{385}$	117.397
3	2	3	3	3	$\frac{37316}{300}$	124.387	3	2	4	1	1	$\frac{15531}{130}$	119.469
3	2	4	1	2	$\frac{31603}{273}$	115.762	3	2	4	1	3	$\frac{12339}{110}$	112.173
3	2	4	2	1	$\frac{13465}{112}$	120.223	3	2	4	2	2	$\frac{92663}{776}$	119.411
3	2	4	2	3	$\frac{13661}{114}$	119.833	3	2	4	3	1	$\frac{45583}{385}$	118.397

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
3	2	4	3	2	$\frac{17244}{143}$	120.587	3	2	4	3	3	$\frac{59763}{495}$	120.733
3	2	5	1	1	$\frac{75687}{629}$	120.329	3	2	5	1	2	$\frac{75687}{629}$	120.329
3	2	5	1	3	$\frac{31876}{273}$	116.762	3	2	5	2	1	$\frac{31110}{251}$	123.944
3	2	5	2	2	$\frac{56366}{466}$	120.957	3	2	5	2	3	$\frac{12669}{110}$	115.173
3	2	5	3	1	$\frac{60258}{495}$	121.733	3	2	5	3	2	$\frac{28227}{234}$	120.628
3	2	5	3	3	$\frac{22821}{189}$	120.746	3	3	1	1	1	$\frac{58594}{536}$	109.317
3	3	1	1	2	$\frac{53570}{466}$	114.957	3	3	1	1	3	$\frac{101924}{902}$	112.998
3	3	1	2	1	$\frac{54502}{466}$	116.957	3	3	1	2	2	$\frac{43658}{385}$	113.397
3	3	1	2	3	$\frac{21687}{189}$	114.746	3	3	1	3	1	$\frac{15661}{130}$	120.469
3	3	1	3	2	$\frac{22065}{189}$	116.746	3	3	1	3	3	$\frac{114759}{971}$	118.186
3	3	2	1	1	$\frac{28213}{247}$	114.223	3	3	2	1	2	$\frac{17126}{149}$	114.940
3	3	2	1	3	$\frac{22098}{195}$	113.323	3	3	2	2	1	$\frac{15646}{133}$	117.639
3	3	2	2	2	$\frac{80387}{664}$	121.065	3	3	2	2	3	$\frac{19106}{161}$	118.671
3	3	2	3	1	$\frac{37016}{300}$	123.387	3	3	2	3	2	$\frac{30859}{251}$	122.944
3	3	2	3	3	$\frac{48734}{390}$	124.959	3	3	3	1	1	$\frac{20685}{175}$	118.200
3	3	3	1	2	$\frac{54502}{466}$	116.957	3	3	3	1	3	$\frac{21943}{187}$	117.342
3	3	3	2	1	$\frac{81051}{664}$	122.065	3	3	3	2	2	$\frac{13547}{114}$	118.833
3	3	3	2	3	$\frac{12449}{110}$	113.173	3	3	3	3	1	$\frac{111634}{902}$	123.763
3	3	3	3	2	$\frac{31110}{251}$	123.944	3	3	3	3	3	$\frac{56366}{466}$	120.957
3	3	4	1	1	$\frac{28707}{247}$	116.223	3	3	4	1	2	$\frac{91111}{776}$	117.411
3	3	4	1	3	$\frac{56034}{469}$	119.475	3	3	4	2	1	$\frac{16958}{143}$	118.587
3	3	4	2	2	$\frac{22878}{195}$	117.323	3	3	4	2	3	$\frac{37016}{300}$	123.387
3	3	4	3	1	$\frac{23268}{195}$	119.323	3	3	4	3	2	$\frac{12779}{110}$	116.173
3	3	4	3	3	$\frac{51094}{422}$	121.076	3	3	5	1	1	$\frac{22065}{189}$	116.746
3	3	5	1	2	$\frac{21035}{175}$	120.200	3	3	5	1	3	$\frac{13353}{112}$	119.223
3	3	5	2	1	$\frac{59268}{495}$	119.733	3	3	5	2	2	$\frac{23073}{195}$	118.323
3	3	5	2	3	$\frac{13775}{114}$	120.833	3	3	5	3	1	$\frac{22821}{189}$	120.746
3	3	5	3	2	$\frac{37916}{300}$	126.387	3	3	5	3	3	$\frac{23065}{187}$	123.342
3	4	1	1	1	$\frac{12009}{110}$	109.173	3	4	1	1	2	$\frac{96271}{811}$	118.707
3	4	1	1	3	$\frac{58594}{536}$	109.317	3	4	1	2	1	$\frac{79723}{664}$	120.065
3	4	1	2	2	$\frac{55565}{469}$	118.475	3	4	1	2	3	$\frac{55565}{469}$	118.475

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
3	4	1	3	1	$\frac{44428}{385}$	115.397	3	4	1	3	2	$\frac{99515}{811}$	122.707
3	4	1	3	3	$\frac{60738}{536}$	113.317	3	4	2	1	1	$\frac{18784}{161}$	116.671
3	4	2	1	2	$\frac{18784}{161}$	116.671	3	4	2	1	3	$\frac{47174}{390}$	120.959
3	4	2	2	1	$\frac{30357}{251}$	120.944	3	4	2	2	2	$\frac{113788}{971}$	117.186
3	4	2	2	3	$\frac{60202}{536}$	112.317	3	4	2	3	1	$\frac{55900}{466}$	119.957
3	4	2	3	2	$\frac{13661}{114}$	119.833	3	4	2	3	3	$\frac{110732}{902}$	122.763
3	4	3	1	1	$\frac{36116}{300}$	120.387	3	4	3	1	2	$\frac{13129}{112}$	117.223
3	4	3	1	3	$\frac{15401}{130}$	118.469	3	4	3	2	1	$\frac{17573}{149}$	117.940
3	4	3	2	2	$\frac{48344}{390}$	123.959	3	4	3	2	3	$\frac{17573}{149}$	117.940
3	4	3	3	1	$\frac{13775}{114}$	120.833	3	4	3	3	2	$\frac{116701}{971}$	120.186
3	4	3	3	3	$\frac{31110}{251}$	123.944	3	4	4	1	1	$\frac{13241}{112}$	118.223
3	4	4	1	2	$\frac{60202}{536}$	112.317	3	4	4	1	3	$\frac{31603}{273}$	115.762
3	4	4	2	1	$\frac{12559}{110}$	114.173	3	4	4	2	2	$\frac{27525}{234}$	117.628
3	4	4	2	3	$\frac{22254}{189}$	117.746	3	4	4	3	1	$\frac{21560}{175}$	123.200
3	4	4	3	2	$\frac{21560}{175}$	123.200	3	4	4	3	3	$\frac{101948}{811}$	125.707
3	4	5	1	1	$\frac{81051}{664}$	122.065	3	4	5	1	2	$\frac{109830}{902}$	121.763
3	4	5	1	3	$\frac{30608}{251}$	121.944	3	4	5	2	1	$\frac{27759}{234}$	118.628
3	4	5	2	2	$\frac{76945}{629}$	122.329	3	4	5	2	3	$\frac{27759}{234}$	118.628
3	4	5	3	1	$\frac{102759}{811}$	126.707	3	4	5	3	2	$\frac{113438}{902}$	125.763
3	4	5	3	3	$\frac{16311}{133}$	122.639	3	5	1	1	1	$\frac{20335}{175}$	116.200
3	5	1	1	2	$\frac{54627}{469}$	116.475	3	5	1	1	3	$\frac{58594}{536}$	109.317
3	5	1	2	1	$\frac{47564}{390}$	121.959	3	5	1	2	2	$\frac{97893}{811}$	120.707
3	5	1	2	3	$\frac{13319}{114}$	116.833	3	5	1	3	1	$\frac{19267}{161}$	119.671
3	5	1	3	2	$\frac{58278}{495}$	117.733	3	5	1	3	3	$\frac{91887}{776}$	118.411
3	5	2	1	1	$\frac{17126}{149}$	114.940	3	5	2	1	2	$\frac{47174}{390}$	120.959
3	5	2	1	3	$\frac{73800}{629}$	117.329	3	5	2	2	1	$\frac{19106}{161}$	118.671
3	5	2	2	2	$\frac{49406}{422}$	117.076	3	5	2	2	3	$\frac{80387}{664}$	121.065
3	5	2	3	1	$\frac{16958}{143}$	118.587	3	5	2	3	2	$\frac{30859}{251}$	122.944
3	5	2	3	3	$\frac{22878}{195}$	117.323	3	5	3	1	1	$\frac{57288}{495}$	115.733
3	5	3	1	2	$\frac{79723}{664}$	120.065	3	5	3	1	3	$\frac{31330}{273}$	114.762
3	5	3	2	1	$\frac{49828}{422}$	118.076	3	5	3	2	2	$\frac{30608}{251}$	121.944

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Table 1: APNACASIC Use Case #1 Experimental Dataset – *Continued*

M	U	V	C	R	F (RAC)	F (float)	M	U	V	C	R	F (RAC)	F (float)
3	5	3	2	3	$\frac{22317}{187}$	119.342	3	5	3	3	1	$\frac{23073}{195}$	118.323
3	5	3	3	2	$\frac{40226}{331}$	121.529	3	5	3	3	3	$\frac{57441}{469}$	122.475
3	5	4	1	1	$\frac{75058}{629}$	119.329	3	5	4	1	2	$\frac{15646}{133}$	117.639
3	5	4	1	3	$\frac{30357}{251}$	120.944	3	5	4	2	1	$\frac{16958}{143}$	118.587
3	5	4	2	2	$\frac{92663}{776}$	119.411	3	5	4	2	3	$\frac{61274}{536}$	114.317
3	5	4	3	1	$\frac{40557}{331}$	122.529	3	5	4	3	2	$\frac{49514}{390}$	126.959
3	5	4	3	3	$\frac{40557}{331}$	122.529	3	5	5	1	1	$\frac{114759}{971}$	118.186
3	5	5	1	2	$\frac{13547}{114}$	118.833	3	5	5	1	3	$\frac{22683}{195}$	116.323
3	5	5	2	1	$\frac{57441}{469}$	122.475	3	5	5	2	2	$\frac{17871}{149}$	119.940
3	5	5	2	3	$\frac{17101}{143}$	119.587	3	5	5	3	1	$\frac{18169}{149}$	121.940
3	5	5	3	2	$\frac{23065}{187}$	123.342	3	5	5	3	3	$\frac{109140}{902}$	120.998

Note there are two observations of F for every combination of $(M, T_{\min}, T_{\max}, C, R)$.

4. Analysis Of Variance (ANOVA)

When processed through the APNACASIC directed by the embedded control system, the following analysis of variance is completed on the fully-factorial design available from the dataset. Each step in the analysis, and the results of each step, are described in the text directly below the table section where the step is shown.

The step number refers to the process of accepting or rejecting a term in the analysis model based on the significance of the F-test statistic calculated from the dataset (within the APNACASIC using the nomographic layers corresponding to the particular F-test degrees of freedom and the F ratio value. These results are presented herein as floating-point values in human-readable format, whereas the (exact) calculations are performed within the APNACASIC as RAC values.

The "#" column represent the term number (from 1 to $2^n - 1$) for n-many independent variables, which means, in this particular example model, $n = 5$ so that the terms are numbered 1 to $2^5 = 32$, numbered in order of the main factors followed by the higher-and-higher order interaction terms; the last (2^n) term always refers to the error term (ϵ). The number associated with any term in the model may be found when the term in question is first considered in the analysis of variance.

The term name follows the "#" column, followed by the Model Acceptance columns: "E" represents an exact F test that is either accepted "In" the model (due to the significance of that term's contribution to explaining the variance of F) or rejected "Out" of the model (for the same reasons). Additionally, "A" represents an accept/reject decision based on an approximate[2] F test statistic, and "U" represents an

[2] The APNACASIC may be coded to perform any particular approximate F test as required. The results found in this document use the *Modified Crump-Satterthwaite-Welch Approximate F Test Statistic*. See the references to Samuel Lee Crump (1919-1963) [*The Estimation of Variance Components in Analysis of Variance*, Biometrics Bulletin (1946), Vol. 2, No. 1, pp. 7-11], Franklin Eves Satterthwaite (1914-?) [*An Approximate Distribution of Estimates of Variance Components*, Biometrics Bulletin (1946), Vol. 2, No. 6, pp. 110-114], and Bernard Lewis Welch (1911-1989) [*The Generalization of 'Student's' Problem When Several Different Population Variances Are Involved*, Biometrica (1947), Vol. 34, No. 1/2, pp. 28-35]. These works were early (1946-7, approximately 20 years after the first publication of the seminal *Statistical Methods for Research Workers* in 1925 by Sir Ronald Aylmer Fisher (1890-1962)), yet pioneering use of the methods found in this memorandum for forming approximate F test statistics based on a judiciously chosen chi-square distribution whose degrees of freedom is based on the data. The term "approximate" refers to the inexact nature of the chi-square approximation to a linear sum of independent

untestable term in the model, usually due to a lack of data (zero degrees of freedom for testing), or due to inappropriate nesting within the model.

The EMS Test column provides the numerator and denominator of the appropriate EMS F-test statistic based on the EMS expansion (see the Summary section). The degrees of freedom for the numerator and denominator then follow, along with the F ratio value in ratio and decimal formats. Finally, the significance column shows the inverse F-test statistic cumulative function value for the calculated F ratio value (provided by the APNACASIC nomographic grid stored linkages for the inverse function). If the significance column shows a value greater than the required significance (90% for this use case), then the term is declared to be "In" the model; otherwise that term is declared to be "Out" of the model (and not further considered).

All terms are considered in the model unless and until this analysis of variance process finds that it should be "out" of the model. Whenever a term in the model is assessed by significance to be "In" the model, consideration continues with the next term (in reverse numerical order). However, whenever a term in the model is assessed by significance to be "Out" of the model, processing starts all over again with the highest numbered term still in the model. The reason for this "restart" comes from the consolidation of the sum of squares into the F-test denominator term sum of squares whenever a model term is eliminated. This means a term that has been assessed by significance to be "In" the model at one point may be further assessed to be "Out" of the model later in the analysis of variance. Only those terms assessed by significance to be "In" the model after all terms have been assessed by significance shall be considered the final analysis of variance, i.e., the unique partitioning of the variance of the dependent term.

All calculations in floating-point format have 6-decimal place precision. If the significance for a term is less than 10^{-6}, the term "$-\inf$ is shown in the significance column; when it is greater than 0.999999, the term "inf is shown.

To review, the required level of significance for a term to be accepted in the model is set (statically) at 90%, and all F tests are exact if allowed by the Expected Mean Squares (EMS) expression (see also the ANOVA Summary section); otherwise an approximate F test is performed.

A running commentary of the ANOVA progress is found at the end of each disposition report page.

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chi-square random variables, and to the approximate nature of the calculated degrees of freedom. The term "modified" refers to the strict use of +1 as a coefficient to form the approximate chi-square linear combinations in both the numerator and denominator of the F test statistic based on the Mean Square values calculated from the data.

APNACASIC EMS Table Disposition Report (Section 1 Of 3)

Step	#	Term	Model In	Model Out	EMS Test	Test df:df	F Ratio	Ratio Value	Signf
1	31	MUVCR		E	$MUVCR : \epsilon$	128 : 675	3.214416/3.433994	0.936057	– inf
		Term MUVCR (#31) Consolidated Into ϵ [New SSE/df = 2729.391357/803].							
2	30	UVCR		E	$UVCR : \epsilon$	64 : 803	2.933408/3.398993	0.863023	– inf
		Term UVCR (#30) Consolidated Into ϵ [New SSE/df = 2917.129395/867].							
3	29	MVCR		E	$MVCR : \epsilon$	32 : 867	2.810543/3.364625	0.835321	0.271845
		Term MVCR (#29) Consolidated Into ϵ [New SSE/df = 3007.066895/899].							
4	28	MUVR		E	$MUVR : \epsilon$	64 : 899	1.766405/3.344902	0.528089	– inf
		Term MUVR (#28) Consolidated Into ϵ [New SSE/df = 3120.116699/963].							
5	27	MUVC		E	$MUVC : \epsilon$	64 : 963	3.275853/3.239997	1.011067	– inf
		Term MUVC (#27) Consolidated Into ϵ [New SSE/df = 3329.771240/1027].							
6	26	MUCR	E		$MUCR : \epsilon$	32 : 1027	4.528567/3.242231	1.396744	0.928565
7	25	VCR		E	$VCR : \epsilon$	16 : 1027	3.547111/3.242231	1.094034	0.644478
		Term VCR (#25) Consolidated Into ϵ [New SSE/df = 3386.524902/1043].							
8	26	MUCR	E		$MUCR : \epsilon$	32 : 1043	4.528567/3.246908	1.394732	0.927767
9	24	UVR		E	$UVR : \epsilon$	32 : 1043	1.258761/3.246908	0.387680	0.000784
		Term UVR (#24) Consolidated Into ϵ [New SSE/df = 3426.805176/1075].							
10	26	MUCR	E		$MUCR : \epsilon$	32 : 1075	4.528567/3.187726	1.420626	0.938337
11	23	UVC		E	$UVC : \epsilon$	32 : 1075	3.017095/3.187726	0.946472	0.446404
		Term UVC (#23) Consolidated Into ϵ [New SSE/df = 3523.352295/1107].							
12	26	MUCR	E		$MUCR : \epsilon$	32 : 1107	4.528567/3.182793	1.422828	0.939270
13	22	UCR		E	$UCR : \epsilon$	16 : 1107	4.035120/3.182793	1.267792	0.789863
		Term UCR (#22) Consolidated Into ϵ [New SSE/df = 3587.914307/1123].							
14	26	MUCR	E		$MUCR : \epsilon$	32 : 1123	4.528567/3.194937	1.417420	0.937280
15	21	MVR		E	$MVR : \epsilon$	16 : 1123	0.668236/3.194937	0.209155	0.000361
		Term MVR (#21) Consolidated Into ϵ [New SSE/df = 3598.605957/1139].							
16	26	MUCR	E		$MUCR : \epsilon$	32 : 1139	4.528567/3.159443	1.433343	0.943179
17	20	MVC		E	$MVC : \epsilon$	16 : 1139	2.326011/3.159443	0.736209	0.241744
		Term MVC (#20) Consolidated Into ϵ [New SSE/df = 3635.822021/1155].							
18	26	MUCR	E		$MUCR : \epsilon$	32 : 1155	4.528567/3.147898	1.438600	0.945051
19	19	MUV		E	$MUV : \epsilon$	32 : 1155	1.219819/3.147898	0.387503	0.000774

For the first 19 steps, only one term, $MUCR$, is accepted by significance (Step 6). This acceptance survives throughout the 19 steps even though the denominator degrees of freedom for its F-test statistic grows from $1,027$ in Step 6 to $1,155$ in Step 18. Note, however, had the required significance for acceptance been, say, 95% rather than 90%, then $MUCR$ would have been rejected by significance, even if the analysis had started at Step 18.

APNACASIC EMS Table Disposition Report (Section 2 Of 3)

Step	#	Term	Model In	Model Out	EMS Test	Test df:df	F Ratio	Ratio Value	Signf
					Term MUV (#19) Consolidated Into ϵ [New SSE/df = 3674.856201/1187].				
20	26	MUCR	E		$MUCR : \epsilon$	32 : 1187	4.528567/3.095919	1.462754	0.952873
21	17	MUC		E	$MUC : \epsilon$	16 : 1187	1.382343/3.095919	0.446505	0.030044
					Term MUC (#17) Consolidated Into ϵ [New SSE/df = 3696.973633/1203].				
22	26	MUCR	E		$MUCR : \epsilon$	32 : 1203	4.528567/3.073128	1.473601	0.956063
23	15	VR		E	$VR : \epsilon$	8 : 1203	0.675416/3.073128	0.219781	0.012525
					Term VR (#15) Consolidated Into ϵ [New SSE/df = 3702.376953/1211].				
24	26	MUCR	E		$MUCR : \epsilon$	32 : 1211	4.528567/3.057289	1.481236	0.958189
25	14	VC		E	$VC : \epsilon$	8 : 1211	1.085040/3.057289	0.354903	0.056175
					Term VC (#14) Consolidated Into ϵ [New SSE/df = 3711.057373/1219].				
26	26	MUCR	E		$MUCR : \epsilon$	32 : 1219	4.528567/3.044346	1.487534	0.959873
27	13	UV		E	$UV : \epsilon$	16 : 1219	1.313983/3.044346	0.431614	0.025336
					Term UV (#13) Consolidated Into ϵ [New SSE/df = 3732.081055/1235].				
28	26	MUCR	E		$MUCR : \epsilon$	32 : 1235	4.528567/3.021928	1.498569	0.962692
29	12	UR	E		$UR : \epsilon$	8 : 1235	7.478863/3.021928	2.474865	0.988424
30	11	UC		E	$UC : \epsilon$	8 : 1235	2.161405/3.021928	0.715241	0.321723
					Term UC (#11) Consolidated Into ϵ [New SSE/df = 3749.372314/1243].				
31	26	MUCR	E		$MUCR : \epsilon$	32 : 1243	4.528567/3.016390	1.501320	0.963375
32	12	UR	E		$UR : \epsilon$	8 : 1243	7.478863/3.016390	2.479409	0.988580
33	10	MV	E		$MV : \epsilon$	8 : 1243	6.682433/3.016390	2.215375	0.975909
34	9	MU		E	$MU : \epsilon$	8 : 1243	4.301177/3.016390	1.425936	0.819064
					Term MU (#9) Consolidated Into ϵ [New SSE/df = 3783.781738/1251].				
35	26	MUCR	E		$MUCR : \epsilon$	32 : 1251	4.528567/3.024606	1.497242	0.962401
36	12	UR	E		$UR : \epsilon$	8 : 1251	7.478863/3.024606	2.472674	0.988356
37	10	MV	E		$MV : \epsilon$	8 : 1251	6.682433/3.024606	2.209357	0.975508
38	7	MC	E		$MC : \epsilon$	4 : 1251	446.358032/3.024606	147.5756	inf
39	6	CR	E		$CR : \epsilon$	4 : 1251	11.889482/3.024606	3.930920	0.996453
40	5	V		E	$V : \epsilon$	4 : 1251	1.625199/3.024606	0.537326	0.291655
					Term V (#5) Consolidated Into ϵ [New SSE/df = 3790.282471/1255].				
41	26	MUCR	E		$MUCR : \epsilon$	32 : 1255	4.528567/3.020145	1.499453	0.962945
42	12	UR	E		$UR : \epsilon$	8 : 1255	7.478863/3.020145	2.476325	0.988478

The pattern of rejecting terms by significance (using the error term as denominator) continues through Step 27. At this point, sixteen terms have been eliminated from the model by significance. However, at Step 29, UR is found to be significant as well as $MUCR$. We also have MV significant at Step 33 and both MC and CR at Steps 38 and 39, respectively, to make five terms found significant in the model. However, V (the first main factor to be considered) is rejected by significance in Step 40. In fact, this factor is not even close to significant (0.291655 versus 0.90). This means V (code for $T_{\max}$) does not explain very much of the variation of F (code for $F_{\max}$), at least in comparison to the other sources available in the data.

APNACASIC EMS Table Disposition Report (Section 3 Of 3)

Step	#	Term	Model In	Model Out	EMS Test	Test df:df	F Ratio	Ratio Value	Signf
43	10	MV	E		$MV : \epsilon$	8 : 1255	6.682433/3.020145	2.212620	0.975731
44	7	MC	E		$MC : \epsilon$	4 : 1255	446.358032/3.020145	147.7935	inf
45	6	CR	E		$CR : \epsilon$	4 : 1255	11.889482/3.020145	3.936725	0.996490
46	4	U	E		$U : \epsilon$	4 : 1255	278.071808/3.020145	92.07233	0.999998
47	1	C	E		$C : \epsilon$	2 : 1255	36102.292969/3.020145	11953.83	0.999989
48	18	MUR		E	$MUR : MUCR$	16 : 32	3.752583/4.528567	0.828647	0.353369
Term MUR (#18) Consolidated Into MUCR (#26) [New SS/df = 204.955475/48].									
49	26	MUCR	E		$MUCR : \epsilon$	48 : 1255	4.269906/3.020145	1.413808	inf
50	12	UR	E		$UR : \epsilon$	8 : 1255	7.478863/3.020145	2.476325	0.988478
51	10	MV	E		$MV : \epsilon$	8 : 1255	6.682433/3.020145	2.212620	0.975731
52	7	MC	E		$MC : \epsilon$	4 : 1255	446.358032/3.020145	147.7935	1.000004
53	6	CR	E		$CR : \epsilon$	4 : 1255	11.889482/3.020145	3.936725	0.996490
54	4	U	E		$U : \epsilon$	4 : 1255	278.071808/3.020145	92.07233	0.999998
55	1	C	E		$C : \epsilon$	2 : 1255	36102.292969/3.020145	11953.83	0.999989
56	16	MCR	E		$MCR : MUCR$	8 : 48	12.421415/4.269906	2.909061	0.990045
57	8	MR	E		$MR : MCR$	4 : 8	179.949997/12.421415	14.48707	0.999023
Approximate F Test Statistic M+ϵ (3) : MC [#7]+MV [#10] (5) Formed.									
58	2			A	$M + \epsilon : MC + MV$	3 : 5	7.339262/90.608093	0.081000	0.032477
Approximate F Test Statistic R+ϵ (4) : CR [#6]+UR [#12] (9) Formed.									
59	3			A	$R + \epsilon : CR + UR$	4 : 9	3.811399/2.152038	1.771065	0.781327

Step 46 finds U is accepted by significance (strongly) and Step 47 finds C is accepted by significance (also strongly). The final two exactly-tested model terms are MCR and MR, which are accepted by significance in Steps 56 and 57, respectively. Step 58 addresses the main factor M. It must be tested approximately by the expression found in the EMS Test column, and it is found to be profoundly insignificant in explaining the variance of F. Finally, Step 59 addresses the main factor R, also as an approximate test. Its significance is much better than for M, however, it is still rejected by significance.

5. ANOVA Summary

At this point we may summarize the terms of the model that were accepted by significance (the “In” terms) in the following table. Note the table also includes approximate test results, even when the terms have been rejected by significance, since the “approximate,” i.e., subjective test results may have to be interpreted in context of the model terms that have been accepted by significance.

Source	**df**	**SS**	**MS**	**F**	**T***	**(N):D****	**Signf**	**EMS**
C_l	2	72204.59	36102.29	11953.83	E	$- : \epsilon$	0.999989	$\sigma_\epsilon^2 + 450\sigma_C^2$
M_i	2	37.995281	18.997641	0.081000	A	$M + \epsilon\ (3) : MC + MV\ (5)$	0.032477	$\sigma_\epsilon^2 + 90\sigma_{MV}^2 + 150\sigma_{MC}^2 + 450\phi_M$
R_m	2	24.450901	12.225451	1.771065	A	$R + \epsilon\ (4) : CR + UR\ (9)$	0.781327	$\sigma_\epsilon^2 + 90\sigma_{UR}^2 + 150\sigma_{CR}^2 + 450\phi_R$
U_j	4	1112.287	278.0718	92.072327	E	$- : \epsilon$	0.999998	$\sigma_\epsilon^2 + 270\sigma_U^2$
CR_{lm}	4	47.557930	11.889482	3.936725	E	$- : \epsilon$	0.996490	$\sigma_\epsilon^2 + 150\sigma_{CR}^2$
MC_{il}	4	1785.432	446.358	147.7935	E	$- : \epsilon$	inf	$\sigma_\epsilon^2 + 150\sigma_{MC}^2$
MR_{im}	4	719.8	179.95	14.48708	E	$- : MCR$	0.999023	$\sigma_\epsilon^2 + 10\sigma_{MUCR}^2 + 50\sigma_{MCR}^2 + 150\phi_{MR}$
MV_{ik}	8	53.459465	6.682433	2.212620	E	$- : \epsilon$	0.975731	$\sigma_\epsilon^2 + 90\sigma_{MV}^2$
UR_{jm}	8	59.830902	7.478863	2.476325	E	$- : \epsilon$	0.988478	$\sigma_\epsilon^2 + 90\sigma_{UR}^2$
MCR_{ilm}	8	99.37132	12.42142	2.909061	E	$- : MUCR$	0.990045	$\sigma_\epsilon^2 + 10\sigma_{MUCR}^2 + 50\sigma_{MCR}^2$
$MUCR_{ijlm}$	48	204.9555	4.269906	1.413808	E	$- : \epsilon$	inf	$\sigma_\epsilon^2 + 10\sigma_{MUCR}^2$
$\epsilon_{n(ijklm)}$	1255	3790.282	3.020145					σ_ϵ^2

* (E) Exact; (A) Approximate – (df); (U) Untestable
** No Numerator Given For Exact Tests

6. ANOVA Conclusions

The two significant main factors are C (the Carbon Percentage) and U (the Minimum Temperature At Exhaust). Between these two factors, we have

$$\frac{SSC + SSU}{SST} = \frac{72204.59 + 1112.287}{80140.012299} \approx 91.49\%$$

so that more than 90% of the variation in F (code for $F_{\max}$) is explained by the variation of those two terms in the (accepted by significance) model. This results is intuitively consistent with experience, as the higher the percentage of carbon, i.e., hydrocarbon compounds in the fuel mixture, the higher the maximum pressure becomes on the piston head during the power stroke. Furthermore, the higher the *minimum* temperature at the end of exhaust stroke becomes, the higher the temperature must have been during the power stroke, i.e., the higher the energy was on the piston head during the just completed power stroke. However, the former effect, namely C, is demonstrated in the data approximately 6 times more strongly than for the latter effect, namely U. Nevertheless, the second-order term of the (accepted by significance) model with the highest sum of squares value is MC (with the only second-order term involving U in the model is UR – involving a main term R deemed not significant – see also the *very* small sum of squares value of CR), thereby reinforcing the conclusion that C is the dominant main factor in the (accepted by significance) model.

Note that V only appears in the (accepted by significance) model in MV (with a very small sum of squares contribution), and that also involves a main factor M that was deemed not significant (even though by approximation only). This appears to mean that the fuel mixture had a consistent upper temperature burn regardless of context; it is only in the minimum temperature variability that a significant contribution to the sum of squares could be found.

From all these facts we may make the conclusion with a high (90%) level of confidence that we may use the linear correlation coefficients of C and U during engine operation to decide when to allow the special fuel additive to be included with the fuel mixture entering the Combustion Cap (and chamber). In fact, the particular composition of the special fuel additive could be fine-tuned, and perhaps separate additives pre-mixed according to particular needs based on the linear correlation coefficients individually and/or collectively.

Including additional higher-order terms from the (accepted by significance) model would reduce the residual error of the linear regression model. However, we already have an approximate 91.49% goodness of fit using only C and U. It might be over-complicating the linear regression calculations to include, say the MC and MR terms, just to improve the goodness of fit percentage by 3.12% (going from 91.49% to 94.61%).

7. Linear Regression Coefficients For C And U Versus F

The analysis of variance has demonstrated that more than 90% of the variation in $F_{\max}$ may be explained by two (main) factors C and $T_{\min}$. Therefore, a linear regression analysis of the coefficients for C and $T_{\min}$ versus $F_{\max}$ will determine a set of circumstances when the fuel additive should be introduced into the fuel mixture to ensure $F_{\max}$ has a high value as consistently as possible. These are the results from the APNACASIC Experiment Mode.

We now consider the functionality of the APNACASIC Surveillance Mode. Define the linear regression model

$$F_{\max} = \mu + \alpha C + \beta T_{\min} + \varepsilon$$

where ε is normally distributed with mean 0 and variance σ^2 (that will be estimated by the (sample) variance of $F_{\max}$ observed in the experimental dataset. Then we have

$$Y_{1350\times 1} = X_{1350\times 2} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}_{2\times 1} + \varepsilon$$

where X is the matrix of $(C, T_{\min})$, one observation for each row of X, and Y is the column vector of $F_{\max}$ values.

Since C had 3 levels, and $T_{\min}$ had 5 levels, we have

$$\left(X^T X\right)^{-1} = \begin{pmatrix} 2\sum_{i=1}^{225}\sum_{j=1}^{3} j^2 & 2\sum_{i=1}^{45}\sum_{j=1}^{3}\sum_{k=1}^{5} jk \\ 2\sum_{i=1}^{45}\sum_{j=1}^{3}\sum_{k=1}^{5} jk & 2\sum_{i=1}^{135}\sum_{j=1}^{5} j^2 \end{pmatrix}^{-1} = \begin{pmatrix} 6300 & 8100 \\ 8100 & 14850 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{11}{20700} & -\frac{1}{3450} \\ -\frac{1}{3450} & \frac{7}{31050} \end{pmatrix}$$

and

$$\widehat{\mu} = \overline{F_{\max}}$$
$$\widehat{\sigma^2} = \frac{1350}{1349}\left(F_{\max} - \overline{F_{\max}}\right)^2$$
$$\begin{pmatrix} \widehat{\alpha} \\ \widehat{\beta} \end{pmatrix} = \left(X^T X\right)^{-1} X^T Y$$

are the least-squares (unbiased) estimators[3] of $\left(\mu, \sigma^2, \alpha, \beta\right)$, where it is critically important that the Y vector is listed in exactly the same order as the X matrix, i.e., the $(C, T_{\min})$ entries correspond exactly to the $F_{\max}$ entries; the order of the $(C, T_{\min})$ entries in X does not matter, as long as it has the same order as the $F_{\max}$ entries in Y.

The results calculated[4] through the APNACASIC are as follows:

$$\widehat{\mu} \approx 109.511$$
$$\widehat{\sigma^2} \approx 59.407$$
$$\begin{pmatrix} \widehat{\alpha} \\ \widehat{\beta} \end{pmatrix} \approx \begin{pmatrix} 29.304 \\ 14.044 \end{pmatrix}$$

[3] Note the ease of calculating a 2×2 matrix inverse to obtain the lease-squares estimators when only two (main) factors were included in the linear regression model. Although the APNACASIC architecture is capable of inverting any size matrix as needed, the fewer the more complicated expressions need be calculated, the quicker results are available for use by the APNACASIC control subsystem, especially when using large experimental or surveillance datasets.

[4] The APNACASIC calculates with RAC values, so that we have, for example, exactly

$$\widehat{\alpha} = \frac{906862075179266931628272001301361821524609021401881782442415835006351746198894874891496712930092922701299519}{31234671899848987002450600424967868667694413228158366875451529979211495153379716076949732430411496318275000}$$

which is the ratio of a 108-digit positive integer (the numerator) with a 107-digit positive integer (the denominator), with similar expressions for the other lease-squares estimators. Since RAC functionality is implemented in the hardware of the APNACASIC, these calculations are performed extremely quickly and are not subject to calculation error. Furthermore, the standard RAC implementation in the APNACASIC provides for the storage and algebraic manipulation of up to 256-digit signed integers, so that calculating these lease-squares estimator values is clearly within typical APNACASIC capabilities.

This means there will be approximately twice the (positive) effect on $F_{\max}$ by increasing C compared to increasing $T_{\min}$. Furthermore, the coefficient of variation given by $\widehat{\frac{\sigma}{\mu}} = \frac{\sqrt{59.407}}{109.511} \approx 7\%$ is low, which means steps taken to increase C by even one-half unit ($\frac{29.304}{2} = 14.652$) (or $T_{\min}$ by one unit) produces a significantly large change in $F_{\max}$ compared to its "natural" variation as found in the experiment dataset.

Therefore, a fuel additive capable of increasing C (the Carbon Proportion In Combustants At Ignition) and $T_{\min}$ (the Minimum Temperature Of Exhaust Achieved At End Of Vent) – and optimally increasing both at a $\frac{29.304}{14.044} \approx 2:1$ ratio – could be added to the fuel mixture whenever, say, the average value of $F_{\max}$ (sensed over 5 seconds) falls below a threshold value for a long enough time, say. more than 2 standard deviations below its average value. In terms of this experiment's results, this would mean the fuel additive(s) would be added to the fuel mixture in a $2:1$ ratio (if multiple additives were available) to increase C and $T_{\min}$, respectively, whenever the 5-second average value of $F_{\max} < 109.511 - 2\sqrt{59.407} = 94.096$.

Note the reasons $F_{\max}$ might decrease are not assessed by this analysis – only "what to do about it" is addressed. Note further that real-time surveillance data of the fuel additive protocol, collected contemporaneously in combustion engines operating over a long-enough time under a variety of environmental conditions, may provide further insight in how to formulate the fuel additive components for even more efficient maintenance of $F_{\max}$ during operations.

8. Epilogue

The APNACASIC may be placed in Experiment Mode at any time if a new (or original) assessment of pertinent factors is needed to optimize a performance statistic. Once the experiment has collected enough data to make a definitive conclusion (based on parameters defined by the APNACASIC designer), it may be placed in Surveillance Mode to carry out the optimization conclusions reached in the immediately prior Experiment Mode. The analytical criteria for moving between these modes are at the discretion of the APNACASIC control subsystem designer.

◇ Timothy Hall
PQI Consulting LLC
382 NE 191st Street PMB 629514
Miami, FL USA 33179-3899
TimothyHall@pqic.tech
http://www.pqic.tech/APNACASIC/UC01.html